CHAPTER 1

Review

1.1 In your own words, what are two differences between product testing and material testing?

Possible answers include: (a) The goal of the two procedures is different. Whereas product testing is design to determine the lifetime of a component under conditions that mimic real-world use, material testing is intended to extract fundamental material properties that are independent of the material's use. (b) The specimen shape is different. Product testing must use the material in the shape in which it will be used in the real product. Material testing uses idealized specimen shapes designed to unambiguously determine one or more properties of the material with the simplest analysis possible.

1.2 What are the distinguishing differences between *elasticity*, *plasticity*, and *fracture*?

Elasticity involves only deformation that is fully reversible when the applied load is removed (even if it takes time to occur). Plasticity is permanent shape change without cracking, even when no load exists. Fracture inherently involves breaking of bonds and the creation of new surfaces. Often two or more of these processes take place simultaneously, but the contribution of each can be separated from the others.

1.3 Write the definitions for engineering stress, true stress, engineering strain, and true strain for loading along a single axis.

$$\sigma_{\text{eng}} = \text{engineering stress} = \frac{\text{load}}{\text{initial cross-sectional area}} = \frac{P}{A_0} \qquad \text{(1-1a)}$$

$$\sigma_{\text{true}} = \text{true stress} = \frac{\text{load}}{\text{instantaneous cross-sectional area}} = \frac{P}{A_i} \qquad \text{(1-2a)}$$

$$\varepsilon_{\text{eng}} = \text{engineering strain} = \frac{\text{change in length}}{\text{initial length}} = \frac{l_f - l_0}{l_0} \qquad \text{(1-1b)}$$

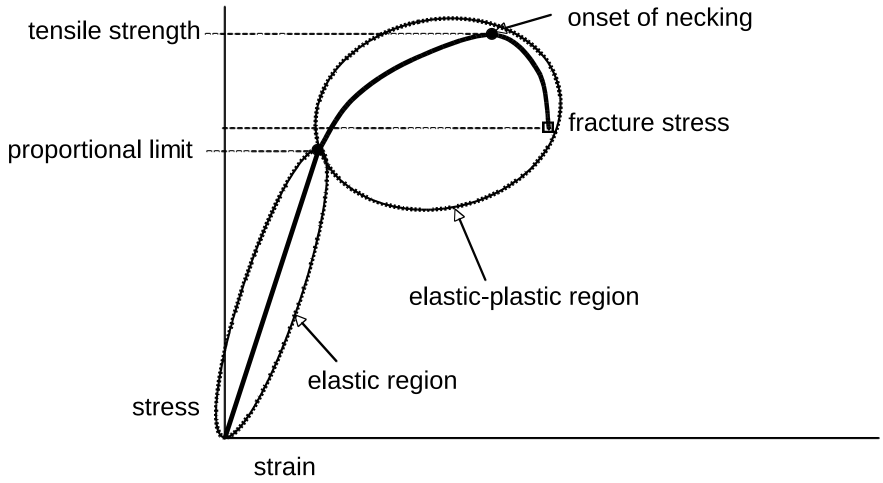
$$\varepsilon_{\text{true}} = \text{true strain} = \ln\frac{\text{final length}}{\text{initial length}} = \ln\frac{l_f}{l_0} \qquad \text{(1-2b)}$$

1.4 Under what conditions is Eq. 1-4 valid? What makes it no longer useful if those conditions are not met?

$$\sigma_{\text{true}} = \frac{P}{A_0}(l_i / l_0) = \sigma_{\text{eng}}(l_i / l_0) = \sigma_{\text{eng}}(1+ \in_{\text{eng}}) \qquad (1\text{-}4)$$

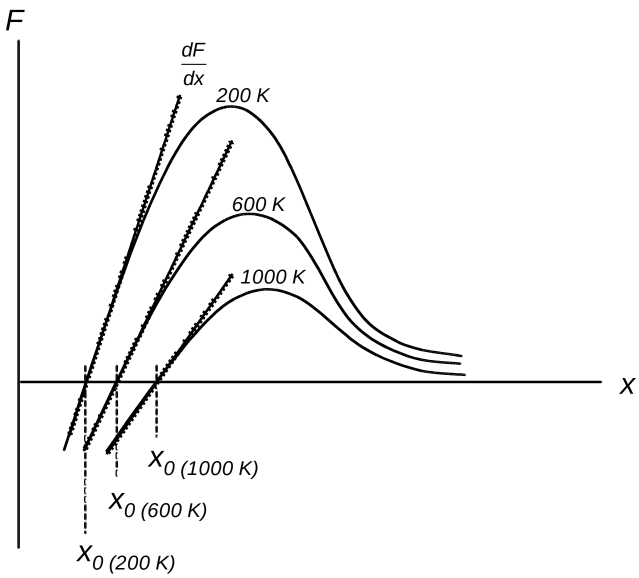
This expression is true when volume is conserved. However, it is only useful if the cross-sectional area is the same everyone on the test specimen. If this isn't the case then the stress and strain will vary from one part of the specimen to another.

1.5 Sketch Figure 1.3, curve 'b' (a ductile metal). Label it with the following terms, indicating from which location on the curve each quantity can be identified or extracted: elastic region, elastic-plastic region, proportional limit, tensile strength, onset of necking, fracture stress.



1.6 On a single set of axes, sketch approximate atomic force vs. atom-separation curves like the one shown in Fig. 1.4b for tungsten at temperatures of 200, 600, and 1000 K. Pay close attention to the point x_0 and the slope dF/dx for each of the curves you draw.

The key features of the plot are the increasing x_0 spacing with increasing temperature (i.e., with thermal expansion) and the decreasing slope associated with decreased elastic modulus. The plot is exaggerated but the trends are reasonable.



1.7 State the critical difference in the processing behavior of *thermoplastics* vs. *thermosets*.

Thermoplastics can be melted and resolidified multiple times, so processing often involves several heating, forming, and cooling steps. Thermosets harden by a one-time chemical reaction so there cannot be any additional forming operations after the cross-linking operation takes place.

1.8 What happens to the stiffness of a polymer as the temperature T_g is exceeded? For what group of polymers is this change the greatest? The smallest?

The stiffness of a polymer decreases above the glass transition temperature, sometimes dramatically. The effect is the largest for amorphous, uncross-linked polymers. It is the smallest for highly cross-linked polymers (such as certain epoxies).

1.9 Write typical values of E for diamond, steel, aluminum, silicate glass, polystyrene, and silicone rubber subjected to small strains (note that the latter value is not included in this chapter, but is widely available). Clearly indicate the units for each value.

The following values are not intended to represent any particular processing method or alloy composition; they are rounded average values for certain material families.

Diamond	*~ 1000 GPa*
Steel	*~ 200 GPa*
Aluminum	*~ 70 GPa*
Silicate glass	*~ 70 GPa*
Polystyrene	*~ 3 GPa*
Silicone rubber	*~ 10 MPa (0.010 GPa)*

1.10 What is the purpose of a *plasticizer*, and what specific effect on room temperature behavior is likely when a plasticizer is added?

A plasticizer is added to a polymer to break up the molecular interactions, allowing more chain mobility than would otherwise be possible for that particular polymer at the temperature of interest. At room temperature, therefore, the polymer is more likely to have a low elastic modulus (i.e., a ordinarily-hard polymer may become flexible).

1.11 Identify a minimum of two structural characteristics and two mechanical characteristics that set *elastomers* apart from other classes of materials (including other polymers).

Elastomers are amorphous and moderately cross-linked. They tend to display significant changes in stiffness as their use temperate exceeds T_g*, but they do not melt at even higher temperature.*

1.12 Define what is meant by *uniaxial, biaxial* and *triaxial* loading.

Uniaxial loading occurs along a single direction, biaxial along two directions, and triaxial along three. Note that there may be multiaxial strains even when the loading is restricted to one or two directions.

1.13 State one advantage and disadvantage of compression testing.

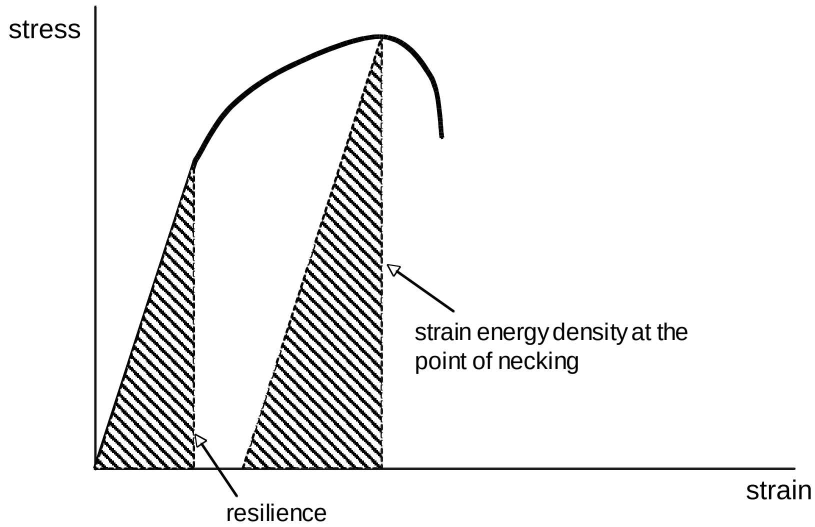
An advantage may be to avoid failure due to tensile cracking at low loads (as in the case for ceramics and glasses), and therefore to allow exploration of degrees of plasticity impossible to achieve under tensile loading. One disadvantage would be the difficulty in achieving ideal friction-free conditions between the specimen and the loading platen.

1.14 Is *buckling failure* initiated by an elastic, plastic, or cracking process? Explain.

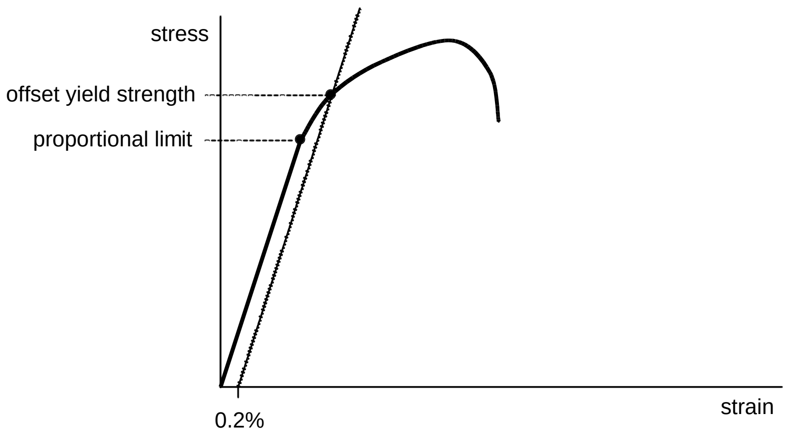
Buckling failure is initially an elastic process in which the member deflects in a direction perpendicular to the loading axis. This failure may then be followed by plasticity or fracture, but these processes are not inherent in buckling.

1.15 What is the difference between the *resilience* and the *strain energy density* of a material under load? Illustrate your answer by reproducing Figure 1.3, curve 'b' (a ductile metal), and annotating it appropriately.

Resilience is a measure of the maximum elastic strain energy stored in the material before the onset of plasticity. The strain energy density is a more general term that is a measure of the stored elastic energy at any point during a mechanical test. It may be greater or less than the resilience, depending on the hardening or softening behavior that takes place after plastic deformation begins.



1.16 Sketch Figure 1.3, curve 'b' (a ductile metal) and show on the figure the difference between the *proportional limit* and the *offset yield strength*.



1.17 Describe when and why bend testing (flexural testing) is most advantageous.

Bend testing may be used for any class of materials. It can be used to assess elastic or plastic properties. It is particularly useful when the material is only available in the shape of a rectangular prism, or when the material would be likely to fail prematurely due to extreme flaw sensitivity (as is usually the case for brittle ceramic and glass materials).

1.18 Where can the maximum stress be found for a rectangular bar undergoing 3-point bending? 4-point bending?

The maximum stress in 3-point bending is found in two locations: at the top and bottom surfaces directly aligned with the central load point. In 4-point bending, the maximum stress is also found on the top and bottom surfaces, but it exists at a constant level between the inner (closer) load points.

1.19 Write the basic isotropic form of Hooke's law relating stress and strain for uniaxial tension/compression loading and shear loading. Define all quantities.

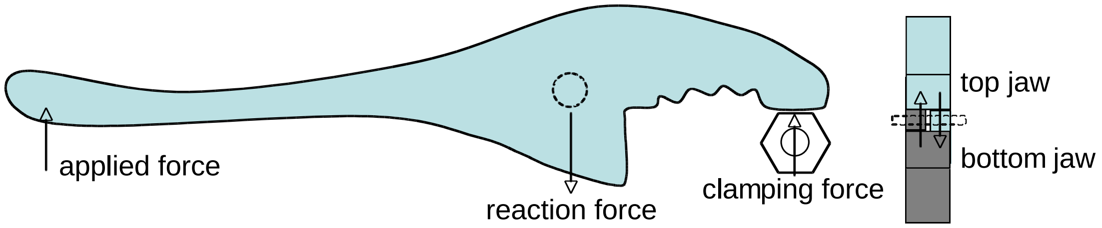
Linear elastic uniaxial tension/compression is described by $\sigma = E \in$, where σ is stress, E is Young's modulus, and $\in$ is strain. An analogous form exists for shear loading, for which $\tau = G\gamma$. In this expression, τ is the shear stress, G is the shear modulus, and γ is the shear strain.

1.20 Why do we define engineering and true stresses for tension/compression loading but not for shear loading?

In tension and compression the cross-sectional area bearing the load changes during deformation, so it is often necessary to account for this change. In shear loading there is distortion of the material but the area over which the force is distributed does not change, so there is no need for a true stress definition.

1.21 Sketch a pair of pliers squeezing an object and use it to show why the hinge pin is under shear loading.

When the clamping force is applied to an object, a reaction force must exist at the pin. The two jaw faces experience equal and opposite clamping forces, so the pin must experience equal and opposite reaction forces at its two ends. These create shear stress within the pin.



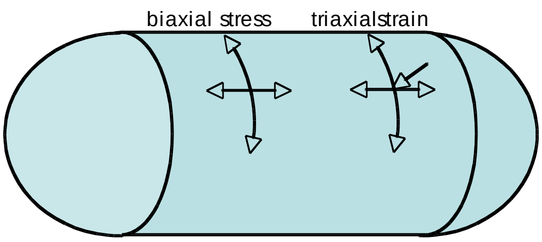
1.22 Write out the most general expression for *tension or compression strain* along a single axis resulting from all possible applied stresses, assuming that the material is elastically isotropic.

$$\varepsilon_{xx} = \frac{1}{E}\sigma_{xx} + \left(\frac{-\nu}{E}\sigma_{yy}\right) + \left(\frac{-\nu}{E}\sigma_{zz}\right) = \frac{\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})}{E}$$

1.23 Write out the most general expression for *shear strain* along a single axis resulting from all possible applied stresses, assuming that the material is elastically isotropic.

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

Sketch and name the *stress state* present in the skin of a cylindrical thin-walled pressure vessel. Repeat for the *strain state*.



1.24 What is the name of the matrix, S_{ij}?

The Compliance Matrix.

1.25 Why can the compliance and stiffness tensors for cubic and orthotropic materials be greatly simplified from the general case?

In cubic and orthotropic materials there are several directions that are structurally identical, and therefore have identical elastic properties. Furthermore, the high degree of symmetry reduces the degree of coupling between applied stresses and induced strains (e.g., the absence of XY, YZ, or XZ shear strains generated by XX, YY, and ZZ stresses).

1.26 Describe the geometric criteria that differentiate *orthotropic* and *cubic* symmetry.

Orthotropic materials have three distinct a, b, and c axes that are each separated by interior angles of $\alpha=\beta=\gamma=90°$. Cubic materials also have three axes separated by $\alpha=\beta=\gamma=90°$, but the three axes are identical.

1.27 Define *hydrostatic stress state*.

A hydrostatic stress state is one in which the three normal stress components, XX, YY, and ZZ, are all equal, and there are no shear stresses.

1.28 What is the primary purpose of the fibers in a composite material? Of the matrix?

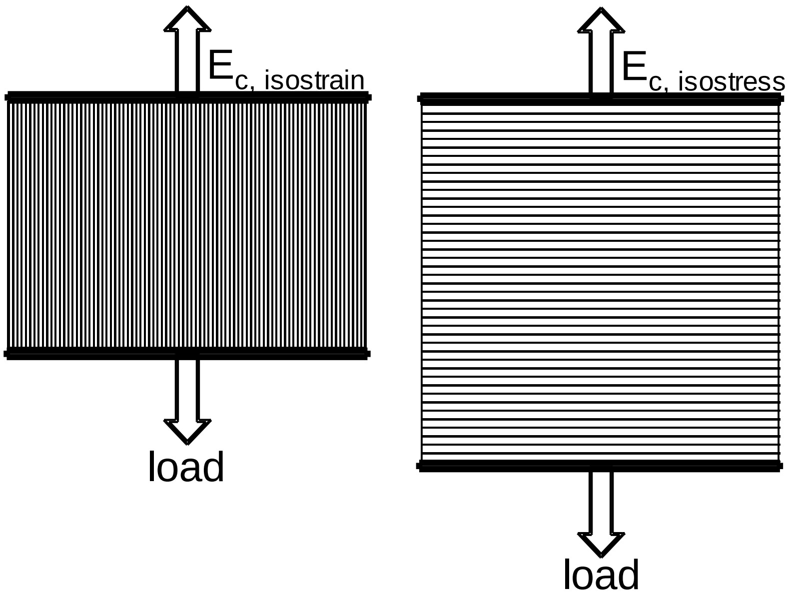
The fibers usually act as the reinforcement phase, supporting the majority of the load. They provide most of the stiffness and stress of the composite material. The matrix hold the fibers together, and serves to transfer the load to the fibers.

1.29 What does it mean for a fiber-reinforced composite to be quasi-isotropic, and how is this typically achieved?

A quasi-isotropic layered composite has elastic properties that are essentially identical in all directions within the plane of the material. This is typically achieved by orienting an equal volume fraction of fibers in each of the 0°, 90°, and ±45° directions.

1.30 Which is the stiffer orientation for a unidirectional fiber-reinforced composite, the isostress orientation or the isostrain orientation? Explain, and provide a sketch to support your answer.

The stiffer direction is the isostrain orientation, in which the fibers and the matrix must strain by equal amounts. In this orientation, the stiffer fibers typically bear the majority of the load, and so the composite stiffness is maximized for a given volume fraction of fibers. In the isostress orientation, the compliant matrix is free to strain to a larger degree than the stiff fibers, so the overall stiffness is lower.

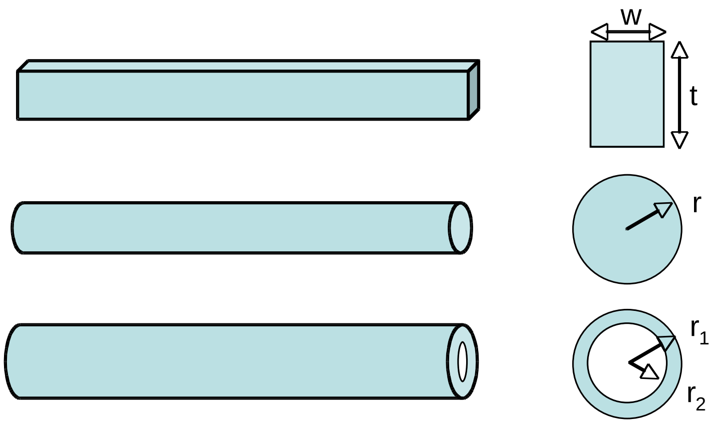


1.31 Why are pairs of materials more likely to experience thermal stress problems when they represent two different material classes?

Pairs of materials are more likely to experience thermal stress problems when they represent two different material classes because the differences in atomic bonding character that exist between material classes can lead to large differences in their coefficients of thermal *expansion. If the materials are joined together, this thermal expansion incompatibility can cause larger thermal stresses to develop.*

Practice

1.32 Sketch a tensile member with (a) a rectangular cross-section, (b) a solid circular cross-section, and (c) a circular tube cross-section, and label the dimensions symbolically (e.g., label the radius for the solid circular case). For each member, write out the definition of engineering stress in terms of the actual dimensions of the component. If the rectangular member has dimensions of width and thickness equal to 1 cm x 0.3 cm, what would be the radius of a solid circular member such that the stress is equal for an equal tensile load? If a tube has an outer radius equal to that of this same solid cylinder, what is the maximum inner radius such that the stress does not exceed 50% of the stress in the solid cylinder?



$$\sigma_{eng} = \frac{F}{A} = \frac{F}{tw}$$

$$\sigma_{eng} = \frac{F}{A} = \frac{F}{\pi r^2}$$

$$\sigma_{eng} = \frac{F}{A} = \frac{F}{\pi r_1^2 - \pi r_2^2} = \frac{F}{\pi\left(r_1^2 - r_2^2\right)}$$

For the rectangular bar, $A_{rect} = (1\,cm)(0.3\,cm) = 0.3\,cm^2$.

To create equal stress under an identical load, the solid cylinder must have the same cross sectional area, so

$$A_{rect} = A_{cyl} = \pi r^2$$

which requires a radius of approximately 0.309 cm.

The tube can have a load-bearing cross section that is equal to half that of the solid cylinder since the maximum stress is half. Setting the outer radius r_1 equal to 0.309 cm and the tube area equal to 0.5(0.3 cm^2) gives the inner radius r_2=0.218 cm.

1.33 A commercially-pure copper wire originally 10.00 m long is pulled until its final length is 10.10 m. It is annealed, then pulled again to a final length of 10.20 m. What is the engineering strain associated with each of the two steps in the process? What is the true strain for each step? What are the total engineering and true strains for the combined steps? Finally, what agreement (if any) is there between the total strains calculated as the sum of two steps of 0.10 m vs. a single step of 0.20 m?

The engineering strain for each step is:

$$1:\quad \varepsilon_{eng1} = \frac{\Delta L}{L_0} = \frac{0.1m}{10\,m} = 0.010 \quad and \quad \varepsilon_{true1} = \ln\left(\frac{L_{final}}{L_0}\right) = \ln\left(\frac{10+0.1m}{10\,m}\right) = 0.00995$$

$$2:\quad \varepsilon_{eng2} = \frac{\Delta L}{L_1} = \frac{0.1m}{10.1m} = 0.0099 \quad and \quad \varepsilon_{true2} = \ln\left(\frac{L_{final}}{L_1}\right) = \ln\left(\frac{10.1+0.1m}{10.1m}\right) = 0.00985$$

So the sum of the two steps in each case is

$$\varepsilon_{eng} = \varepsilon_{eng1} + \varepsilon_{eng2} = 0.010 + 0.0099 = 0.0199 \quad and$$
$$\varepsilon_{true} = \varepsilon_{true1} + \varepsilon_{true2} = 0.00995 + 0.00985 = 0.0198$$

Calculating the total strain as if the elongation were performed in a single step:

$$\varepsilon_{eng} = \frac{\Delta L}{L_0} = \frac{0.2\,m}{10\,m} = 0.020 \quad and \quad \varepsilon_{true} = \ln\left(\frac{L_{final}}{L_0}\right) = \ln\left(\frac{10+0.2\,m}{10\,m}\right) = 0.01980$$

So, it can be seen that the sum of the individual engineering strains does not agree with the strain calculated for a single elongation step of 0.20 m, whereas the total true strains is the same regardless of whether the 0.20 m elongation is applied in one increment or two.

1.34 A 3-mm-long gold alloy wire intended to electrically bond a computer chip to its package has an initial diameter of 30 μm. During testing, it is pulled axially with a load of 15 grams-force. If the wire diameter decreases uniformly to 29 μm, compute the following:
(a) The final length of the wire.
(b) The true stress and true strain at this load.
(c) The engineering stress and strain at this load.

The final length of the wire is $l_f = l_0\left(\frac{A_0}{A_f}\right) = (3\,mm)\left(\frac{\pi\left(\frac{30\times10^{-6}\,m}{2}\right)^2}{\pi\left(\frac{29\times10^{-6}\,m}{2}\right)^2}\right) = 3.21mm$

The true stress and true strain are

$$\sigma_{true} = \frac{P}{A_{final}} = \frac{(15\,g-f)\left(\frac{9.807\times10^{-3}\,N}{1g-f}\right)}{\pi\left(\frac{29\times10^{-6}\,m}{2}\right)^2} = 223\,MPa$$

$$\varepsilon_{true} = \ln\left(\frac{L_f}{L_0}\right) = \ln\left(\frac{3.21mm}{3\,mm}\right) = 0.0677$$

The engineering stress and engineering strain are

$$\sigma_{eng} = \frac{P}{A_0} = \frac{(15\,g-f)\left(\frac{9.807\times10^{-3}\,N}{1g-f}\right)}{\pi\left(\frac{30\times10^{-6}\,m}{2}\right)^2} = 208\,MPa$$

$$\varepsilon_{eng} = \frac{L_f - L_0}{L_0} = \frac{3.21-3\,mm}{3\,mm} = 0.070$$

1.35 A cylindrical rod of Ni 200 alloy has the following properties: E = 204 GPa, ν = 0.31. It is loaded *elastically* in compression at 12.5 kN. If the original rod length and diameter are 20 mm and 15 mm, respectively, determine the rod length and diameter under load.

We need to determine the axial strain, then the axial and radial dimensions can be calculated. Because the loading is elastic, we can apply the Young's modulus and Hooke's law as follows

$$\varepsilon_{eng} = \frac{\sigma}{E} = \frac{P}{A_0 E} = \frac{-12.5\times10^3\,N}{\pi\left(\frac{15\times10^{-3}\,m}{2}\right)^2\left(204\times10^9\,\frac{N}{m^2}\right)} = -3.47\times10^{-4}$$

$$\Delta L = L_0\varepsilon_{eng} = (20\,mm)(-3.47\times10^{-4}) = -6.94\times10^{-3}\,mm$$
$$L_f = L_0 + \Delta L = 19.9931\,mm$$

Then relate the axial strain to the radial strain using the Poisson's ratio

$$\varepsilon_{Poisson} = -\nu\varepsilon_{applied} = -0.31(-3.47\times10^{-4}) = 1.0757\times10^{-4}$$

$$\Delta d = d_0\varepsilon = (15\,mm)(1.0757\times10^{-4}) = 1.614\times10^{-3}\,mm$$
$$d_f = d_0 + \Delta d \approx 15.0016\,mm$$

1.36 A 0.5 m long rod of annealed 410 stainless steel was loaded to failure in tension. The rod originally had a square cross section measuring 1.25 cm on a side. What was the load necessary to break the sample? If 85% of the total elongation occurred prior to the onset of localized deformation, compute the true stress at the point of incipient necking.

We need to know the tensile strength (the maximum strength before necking commences) and the total elongation at failure. From Table 1.2a, these values are 515 MPa and 35%, respectively. Thus

$$P = A_0\sigma = (1.25\times10^{-2}\,m)^2(515\times10^6\,Pa) \approx 80.5\,kN$$

To determine the true stress at necking, we need the actual cross sectional area, which can be determined by assuming constant volume deformation throughout the period of uniform elongation.

$$Uniform\ elongation = 0.85(35\%) = 29.75\%$$

so the instantaneous length at necking was 129.75% of the original, or 0.64875 m. Then

$$A_i = \frac{A_0 L_0}{L_i} = \frac{\left(1.5625 \times 10^{-4} m^2\right)\left(0.5\, m\right)}{0.64875\, m} = 1.204 \times 10^{-4} m^2$$

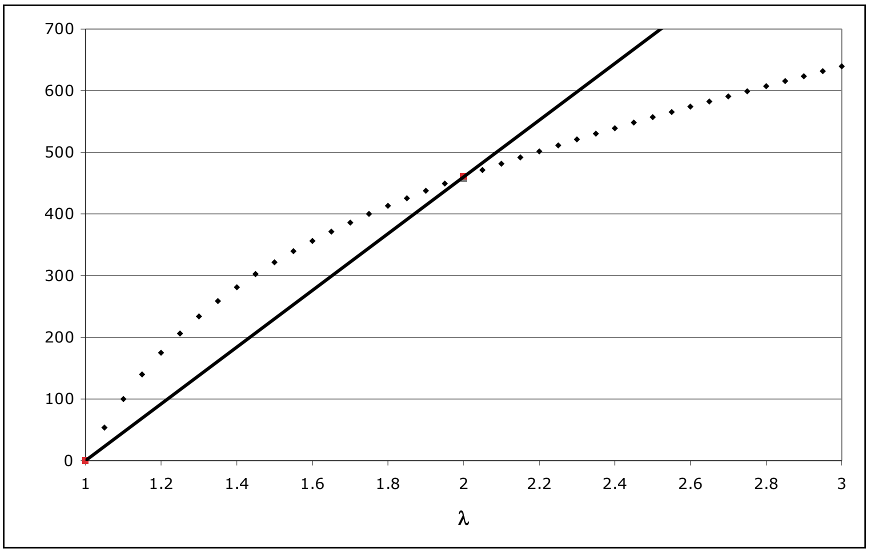
$$\sigma_{true} = \frac{P}{A_i} = \frac{80.5 \times 10^3 N}{1.204 \times 10^{-4} m^2} = 668.6\, MPa$$

1.37 Natural rubber is tested in tension to a maximum extension ratio of λ=3. The Mooney-Rivlin constants for this material are found to be C_1=0.069 MPa and C_2= 0.125 MPa. Plot the corresponding uniaxial stress vs. extension ratio behavior over the tested range. Derive an expression for the slope of the function, then determine the secant and tangent moduli at 100% strain.

Plot equation 1-14 first, then determine the slope of the line between λ=1 and λ=2 (i.e., at 100% strain). The slope is the secant modulus, which is 0.460 MPa for this particular rubber material. To derive an expression for the slope of the function, take the first derivative of stress with respect to extension ratio. The value of the slope at λ=2 is the tangent modulus, which in this case is 0.219 MPa.

$$\sigma = 2\left(C_1 + \frac{C_2}{\lambda}\right)\left(\lambda - \frac{1}{\lambda^2}\right) = 2C_1\lambda + 2C_2 - \frac{2C_1}{\lambda^2} - \frac{2C_2}{\lambda^3}$$

$$\left(\frac{d\sigma}{d\lambda}\right) = 2C_1 + \frac{4C_1}{\lambda^3} + \frac{6C_2}{\lambda^4}$$



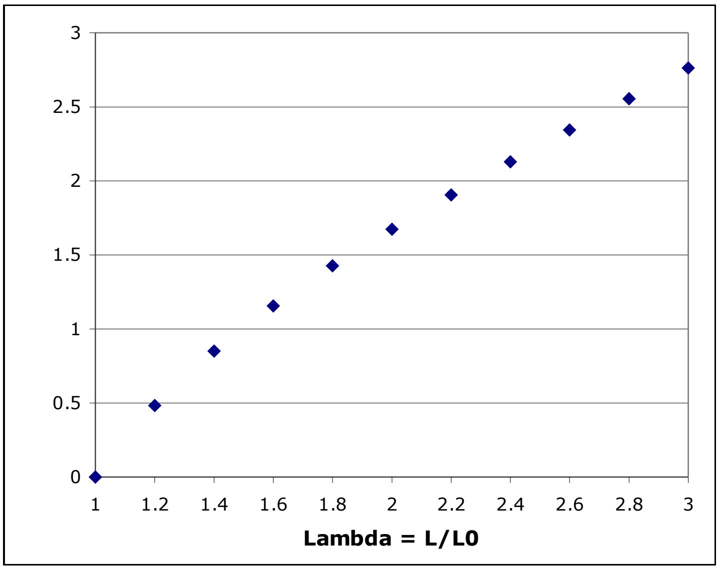
1.38 Compare the resilience of annealed alloy Ti-6Al-4V with that of annealed stainless steel alloy 304. Then compare the elastic strain energy density just prior to the onset of necking for both alloys. Assume that the Young's moduli are 114 GPa for the Ti-6-4 and 193 GPa for the 304 stainless.

Resiliance is the area under the stress-strain curve at the proportional limit, whereas the elastic strain energy density at necking is determined from the elastic strain that exists at that point. From Eq. 1-19

$$\text{resilience} = \frac{\sigma_{\max}^2}{2E}$$

We need the yield strength values from Table 1.2a: for annealed Ti-6Al-4V it is 925 MPa and for annealed 304 stainless it is 240 MPa. From these values, we see that the resilience of the Ti-6-4 is 3.75 MN/m^2 (or MJ/m^3). The resilience of the 304 stainless is 0.149 MN/m^2 (or MJ/m^3). The elastic strain energy density at necking is calculated the same way, but with the tensile strength values: 995 and 565 MPa, respectively, for the Ti-6-4 and the 304 stainless, respectively. The corresponding strain energy densities are 4.34 and 0.827 MN/m^2 (or MJ/m^3). Perhaps surprisingly, even though the stainless steel is much stiffer than the Ti alloy, high yield and tensile strength values make the Ti alloy the better material for elastic energy storage (at least when both are in the annealed state).

1.39 A cylindrical elastomeric rope is used to make a slingshot. The diameter is 15 mm and the original length is 1 m. It is stretched to twice its original length (λ=2) then released. The behavior is fully elastic and not time-dependent over the time span of the slingshot's use. The stress-extension ratio behavior is shown in the plot below.



(a) If the first two data points were used to calculate an initial linear elastic Young's modulus E_0, what would that value be? Answer in GPa or MPa units.

Take the slope of a line passing through the first two data points: E_0=(0.5-0)/(1.2-1)=2.5 N/mm^2, which converts to 2.5 N/10^{-6} m^2 or simply 2.5 MPa.

(b) Based on the plot above, what is the diameter of the rope at λ=2? Noting that this is a large strain, state and justify any assumption you must make to answer this question.

When λ=2, the length is twice the original. Assume that volume is constant during elastic deformation (i.e., ν=0.5), a reasonable decision for rubber materials, so that $A_0 l_0 = A_f l_f$. Then

$$\pi\left(\frac{d_0}{2}\right)^2 l_0 = \pi\left(\frac{d_f}{2}\right)^2 l_f$$

$$d_f = \sqrt{\frac{l_0}{l_f}d_0^2} = \sqrt{\frac{1}{2}d_0^2} = \frac{1}{\sqrt{2}}d_0 = \frac{1}{\sqrt{2}}(15\ mm) = 10.6\ mm$$

Note that if you use the standard linear relationship between axial strain, radial strain, and Poisson's ratio you will get a significantly smaller final diameter. The linear expression is only accurate for small strains, whereas in this case λ=2, so ε=1.00 (definitely a large strain).

(c) For a rubber material, one possible non-linear relationship relating stress and extension ratio is given by Eq 1-13. Assume that this is a reasonable expression for the behavior depicted above, and calculate the expected stored energy density at λ = 2. Be sure to report units.

Eq. 1-13 is $\sigma = \frac{E_0}{3}\left(\lambda - \frac{1}{\lambda^2}\right)$ *and the stored energy density is given by the area under the stress-strain (or stress-extension ratio) plot. Determine this by integrating Eq. 1-13 from λ=1 to λ=2.*

$$SED = \frac{E_0}{3}\int_1^2\left(\lambda - \frac{1}{\lambda^2}\right)d\lambda = \frac{E_0}{3}\left[\frac{1}{2}\lambda^2 + \frac{1}{\lambda}\right]_1^2 = \frac{2.5MPa}{3}\left[\left(\frac{4}{2}+\frac{1}{2}\right)-\left(\frac{1}{2}+1\right)\right] = 0.83\times10^6 J/m^3$$

1.40 A rectangular plate 100 mm long, 10 mm wide and 3 mm thick is formed from fused silica. It is tested in 3-point bending until it fails with a modulus of rupture of 110 MPa at a load of 66 N. Assume the central load point is on the top of the beam.

(a) How far apart must have been the lower supports?

Eq. 1-24 shows that the rupture strength can be related to the specimen dimensions and the rupture load as

$$\sigma_{3\text{-pt.}} = \frac{3PL}{2bh^2}$$

so we can rearrange the expression to solve for L, the support span:

$$L = \frac{2bh^2\sigma_{3\text{-pt.}}}{3P} = \frac{2(0.010\ m)(0.003\ m)^2(110\times10^6\ N/m^2)}{3(66\ N)} = 0.1\ m = 100\ mm$$

(b) What was the maximum stress (magnitude and sign) on the top side of the beam halfway between the central load point and the left-hand lower load point?

If the top side of the beam is associated with the single load point, it will be the compressive side. Stress on the compressive side is negative in sign, is a maximum on the outer surface, and changes linearly from the center load point to the outer load point. Thus the stress halfway along this surface must have been -110 MPa/2 = -55 MPa.

(c) What was the stress (magnitude and sign) directly beneath the central load point exactly 1.5 mm from the top surface?

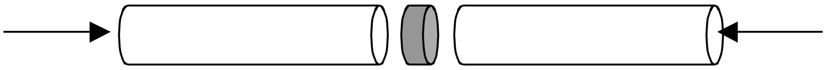
The plane located parallel to the top and bottom surfaces at a distance of 1.5 mm from the top is the central plane of the plate. It is the neutral axis for which there is no length change during bending, so the stress will be zero.

(d) If the same plate was tested in pure tension would the stress at failure probably be higher or lower than measured by three point bending? Why?

In tension a larger fraction of the surface is exposed to the maximum stress, so it is statistically more likely that a fatal flaw will be subjected to sufficiently high stress to cause failure. One would therefore expect that the stress at failure would probably be lower in tension than in three point bending.

1.41 A disk of SBR elastomer 3.0 cm in diameter x 0.5 cm thick is used as a cushioning surface between two steel rods of the same diameter, as shown below (not to scale).

	G	E	ν	SBR/steel interface shear strength	CTE
SBR	3.4 MPa	10 MPa	0.49	2 MPa	220 x 10^{-6} °C^{-1}



(a) If the rods are brought together with an axial force of 100 N such that the SBR is compressed elastically between them, what is the thickness of the SBR under load?

$$\sigma = \frac{P}{A_0} = \frac{P}{\pi\left(\frac{d}{2}\right)^2} = \frac{-100N}{\pi\left(\frac{0.030m}{2}\right)^2} \approx -141.5kPa$$

Then, using the definition of strain, we can say that

$$L_f = L_0 + L_0\varepsilon = L_0 + L_0\left(\frac{\sigma}{E}\right) = 0.5cm + 0.5cm\left(\frac{-141.5\times10^3 Pa}{10\times10^6 Pa}\right) = 0.4929cm$$

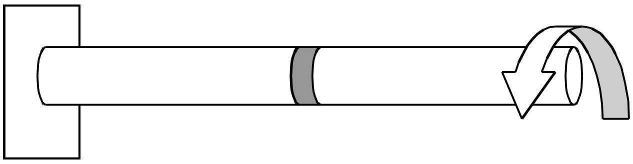
(b) Under the same conditions as part ‘a’, what is the greatest possible diameter of the SBR under load?

Use Poisson’s Ratio to relate axial strain to radial strain. This will give a maximum diameter because the SBR disk is constrained at its ends by the attachment to the rods.

$$\varepsilon_{radial} = -\nu\varepsilon_{axial} = -\nu\left(\frac{\sigma_{axial}}{E}\right) = -0.49\left(\frac{-141.5\times10^3 Pa}{10\times10^6 Pa}\right) = 6.9335\times10^{-3}$$

$$D_f = D_0 + D_0\varepsilon_{radial} = 3cm + 3cm\left(6.9335\times10^{-3}\right) = 3.021\,cm$$

(c) If the SBR is bonded to the rods, how far can one rod be rotated with respect to the other before the SBR/rod interface fractures? Assume that the rods are essentially rigid and the distance between them remains constant. Please answer in degrees of rotation.



From Eq. 1-26 and Eq. 1-29, $\tau = G\gamma$ *and* $\gamma = \tan\theta$*. Therefore*

$$\theta = \tan^{-1}\left(\frac{\tau}{G}\right) = \tan^{-1}\left(\frac{2\ MPa}{3.4\ MPa}\right) \approx 30.5°$$

1.42 A solid cylindrical rod 12 mm in diameter and 50 mm in length is attached to a rigid support at one end and twisted at its free end by 14°. If the Poisson's ratio for this isotropic material is 0.34 and Young's modulus is 70 GPa, what is the maximum shear stress induced?

From Eq. 1-26 and Eq. 1-33, $\tau = G\gamma$ *and* $\gamma_{max} = D\phi/2L$. *We need G, which is available from the other two elastic constants because the material is elastically isotropic. Together we have*

$$\tau_{max} = G\gamma_{max} = \left(\frac{E}{2(1+\nu)}\right)\left(\frac{D\phi}{2L}\right) = \left(\frac{70\ GPa}{2(1+0.34)}\right)\left(\frac{12\ mm(5°)\left(\frac{\pi}{180}\right)}{2(50\ mm)}\right) = (26.12\ GPa)(0.0105) \approx 274\ MPa$$

1.43 Spinel ($MgAl_2O_4$) "optical ceramic" is a transparent polycrystalline ceramic with a combination of high hardness, low density, and optical properties that make it very attractive for fracture resistant windows (e.g., as armor or in a future manned space vehicle). It has the mechanical properties listed below.

	G	**E**	**ν**	**Flexure strength**	**CTE**
Spinel	192 GPa	277 GPa	0.26	200 GPa	7 x 10^{-6} °C^{-1}

(a) A rectangular plate 1 mm x 10 mm x 100 mm is mounted for use as a protective window over a sensor. In the course of mounting, a compressive load of 2.5 kN is exerted along the long (100 mm) axis. There is no constraint along the other two axes. What is the *stress* along the long axis?

$$\sigma = \frac{P}{A_0} = \frac{-2500N}{(0.010m)(0.001m)} = -250\ MPa$$

(b) What is the *strain* along the long axis?

$$\varepsilon = \frac{\sigma}{E} = \frac{-250\ MPa}{277000\ MPa} = -9 \times 10^{-4}$$

Note that this strain is negative because of the applied compressive load.

(c) What is the *strain* along the *width* (10 mm) axis?

$$\varepsilon_{radial} = -\nu\varepsilon_{axial} = (-0.26)(-9 \times 10^{-4}) = 2.35 \times 10^{-4}$$

Note that this strain is positive because the material bulges outward radially in response to the applied axial compressive load..

(d) What is the *strain* along the *thickness* (1 mm) axis?

The same as in the width direction because the material is elastically isotropic.

(e) Now the plate is rigidly constrained along its width (the 10 mm axis). This has the consequence that the plate cannot change length along that axis, although it is still free to change thickness dimension. The same 2.5 kN load is exerted along the long axis. Now what is the *strain* along the *long* axis?

Set up the generalized Hooke's Law equations for the two axes that we care about: the direction of the applied load (arbitrarily call it direction 3) and the constrained direction (direction 2). The material is free to expand or contract in direction 1 (thickness) so the stress along that axis must be zero (i.e., this is a plane stress condition). Beginning with direction 2:

$$\varepsilon_2 = \frac{-\nu}{E}\sigma_1 + \frac{1}{E}\sigma_2 + \frac{-\nu}{E}\sigma_3 = 0$$

$$\sigma_2 = \nu\sigma_3$$

then moving on to direction 3:

$$\varepsilon_3 = \frac{-\nu}{E}\sigma_1 + \frac{-\nu}{E}\sigma_2 + \frac{1}{E}\sigma_3 = \frac{-\nu}{E}(\nu\sigma_3) + \frac{1}{E}\sigma_3$$

$$\varepsilon_3 = \left(\frac{1-\nu^2}{E}\right)\sigma_3 = \left(\frac{1-0.26^2}{277000\ MPa}\right)(-250\ MPa) = -8.4\times 10^{-4}$$

1.44 Compute the moduli of elasticity for nickel and 3C silicon carbide single crystals in the ⟨100⟩, ⟨110⟩, and ⟨111⟩ directions. Compare these values with Young's modulus values reported for polycrystalline samples of Ni and β-SiC (204 GPa and 410 GPa, respectively). Then calculate the relative degree of anisotropy for both materials, and compare it to that of aluminum, spinel, and copper.

First, calculate the modulus in each direction using the direction cosines, as per Eq. 1-55:

$$\frac{1}{E} = S_{11} - 2[(S_{11} - S_{12}) - \tfrac{1}{2}S_{44}](l_1^2 l_2^2 + l_2^2 l_3^2 + l_1^2 l_3^2)$$

Doing so, we find the following:

	E(100) GPa	*E(110) GPa*	*E(111) GPa*	*E(iso) GPa*
Ni	*137.0*	*232.6*	*303.0*	*204*
β-SiC	*270.3*	*421.1*	*517.2*	*410*

From this table it can be seen that the <110> values match reasonably well with the isotropic polycrystalline values. It can also be seen that both materials exhibit fairly large differences between the <100> and the <111> directions, with <111> being stiffer in both cases. Finally, the degree of anisotropy can be calculated using the equation from Table 1.8 and compared to data taken from that same table. We can conclude that both Ni and β-SiC are more anisotropic than Al, similar in anisotropy to spinel, and less anisotropic than Cu.

	$\left[\frac{2(s_{11}-s_{12})}{s_{44}}\right]$
Ni	*2.50*
β-SiC	*2.23*
Al	*1.22*
Spinel	*2.53*
Cu	*3.20*

1.45 Assume the following *elastic* loading exists on a block of copper:

σ_X = 325 MPa, σ_Y = 80 MPa, and τ_Y = 40 MPa

Calculate ε_X and ε_Z for this block assuming

(a) that it is a random polycrystalline material

Random polycrystals can be treated as isotropic materials. Use the Generalized Hooke's Law, for which there is no connection between a shear stress and any of the normal strains.

$$\varepsilon_X = \frac{1}{E}\sigma_X + \frac{-\nu}{E}\sigma_Y + \frac{-\nu}{E}\sigma_Z = \frac{1}{129800\ MPa}\left(325\ MPa - 0.343\left(80\ MPa\right) + 0\right) = 2.29\times10^{-3}$$

$$\varepsilon_Z = \frac{-\nu}{E}\sigma_X + \frac{-\nu}{E}\sigma_Y + \frac{1}{E}\sigma_Z = \frac{1}{129800\ MPa}\left(-0.343\left(325\ MPa\right) - 0.343\left(80\ MPa\right) + 0\right) = -1.07\times10^{-3}$$

(b) that it is a single crystal with the tensile and shear axes lining up along unit cell axes.

In this case, we must use the matrix form of Hooke's Law which also lacks a connection between the shear stress and the normal strains because the material has cubic symmetry:

$$\varepsilon_X = S_{11}\sigma_X + S_{12}\sigma_Y + S_{13}\sigma_Z = S_{11}\sigma_X + S_{12}\sigma_Y + S_{12}\sigma_Z$$

$$\varepsilon_X = \left(1.50\times10^{-11}Pa^{-1}\right)\left(325\times10^{6}\ Pa\right) + \left(-0.63\times10^{-11}Pa^{-1}\right)\left(80\times10^{6}\ Pa\right) + 0 = 4.371\times10^{-3}$$

$$\varepsilon_Z = S_{31}\sigma_X + S_{32}\sigma_Y + S_{33}\sigma_Z = S_{12}\sigma_X + S_{12}\sigma_Y + S_{11}\sigma_Z$$

$$\varepsilon_X = \left(-0.63\times10^{-11}Pa^{-1}\right)\left(325\times10^{6}\ Pa\right) + \left(-0.63\times10^{-11}Pa^{-1}\right)\left(80\times10^{6}\ Pa\right) + 0 = -2.098\times10^{-3}$$

(c) Explain why the relative strain values you calculated along the X axis make sense for the two cases, based on the elastic anisotropy of copper.

The stiffness of copper along X, a {100} direction, is lower than the stiffness of isotropic copper. This means the compliance along this direction is higher for the single crystal than the compliance for isotropic copper. Thus is makes sense that for the same loading there would be a larger strain along X for the anisotropic case.

1.46 A weight lifter holds 300 pounds over his head, supporting the bar with both arms vertical.

(a) What is the stress in each humerus (upper arm bone), assuming that it can be approximated as a solid cylindrical rod with cross sectional area of 1.05 in^2? Use SI units.

$$\sigma = \frac{P}{A_0} = \frac{-300\ lbs.\left(4.448N\ /\ 1\ lb.\right)}{1.05\ in^2\left(645\times10^{-6}m^2\ /\ 1\ in^2\right)} = -1.97\ MPa$$

(b) What are the corresponding axial and radial strains?

Consulting Fig. 1.15, direction 3 is axial and direction 1 is radial. Then using values from Table 1.5 and equation 1-49, we find that

$$\varepsilon_3 = \frac{-\nu_{31}}{E_{11}}\sigma_1 + \frac{-\nu_{32}}{E_{22}}\sigma_2 + \frac{1}{E_{33}}\sigma_3 = 0 + 0 + \frac{1}{20.0\ GPa}(-0.00197\ GPa) = -9.85\times10^{-5}$$

$$\varepsilon_1 = \frac{1}{E_{11}}\sigma_1 + \frac{-\nu_{12}}{E_{22}}\sigma_2 + \frac{-\nu_{13}}{E_{33}}\sigma_3 = 0 + 0 + \frac{-0.22}{20.0\ GPa}(-0.00197\ GPa) = 2.167\times10^{-5}$$

(c) If the humerus is 9 inches long, what are the length and diameter changes associated with this massive load? Please give this answer in inches.

$$\Delta L = L_0\varepsilon = 9\ in\left(-9.85\times10^{-5}\right) = -8.865\times10^{-4}\ in$$

$$\Delta D = \varepsilon D_0\varepsilon = 2\varepsilon\sqrt{\frac{A_0}{\pi}} = 2\left(2.167\times10^{-5}\right)\sqrt{\frac{1.05in^2}{\pi}} = 2.5\times10^{-5}\ in$$

1.47 A thin-walled pressure vessel is subjected to internal pressure such that a hoop stress of 100 MPa develops. Imagine that the vessel is made of an orthotropic continuous fiber composite with most of the fibers running around the circumference. The elastic constants for this material are given below, with direction 3 around the circumference, direction 2 along the length, and direction 1 through the thickness. What is the *strain* in the hoop direction?

	11	**22**	**33**	**44**	**55**	**66**	**12**	**13**	**23**
S (GPa^{-1})	0.083	0.075	0.05	0.161	0.178	0.221	0.031	0.019	0.018
C (GPa)	18.0	20.2	27.6	6.23	5.61	4.52	9.98	10.1	10.7

There are two stresses present, one in the circumferential direction (3) and one in the longitudinal direction (2). The longitudinal stress will be half that of the circumferential stress. Solve for strain along axis 3, using the orthotropic version of the compliance matrix. In order to do so, we must recognize that 31=13 and 32=23 because of symmetry.

$$\varepsilon_3 = S_{31}\sigma_1 + S_{32}\sigma_2 + S_{33}\sigma_3 = S_{13}\sigma_1 + S_{23}\sigma_2 + S_{33}\sigma_3 = 0 + S_{23}\left(0.5\times\sigma_3\right) + S_{33}\sigma_3$$

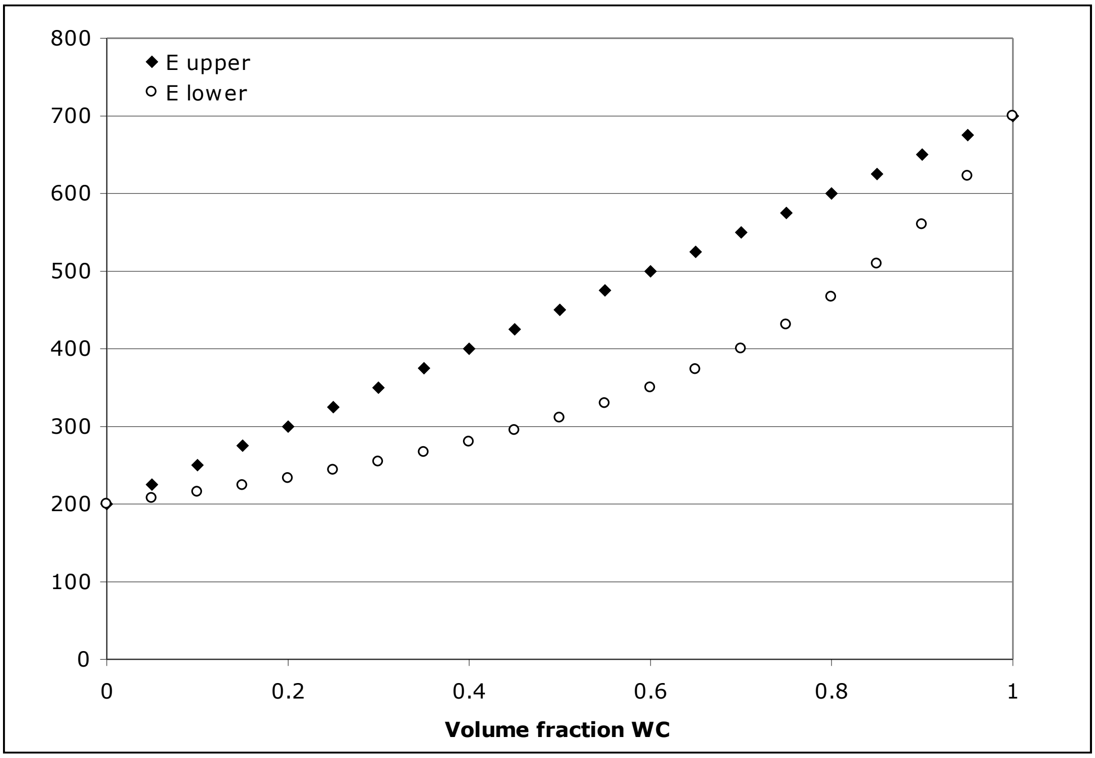
$$\varepsilon_3 = \left(0.018\times10^{-9}Pa\right)\left(50\times10^6Pa\right) + \left(0.05\times10^{-9}Pa\right)\left(100\times10^6Pa\right) = 5.9\times10^{-3}$$

1.48 The mechanical properties of cobalt may be improved by incorporating fine particles of tungsten carbide (WC). Given that the moduli of elasticity of these materials are, respectively, 200 GPa and 700 GPa, plot modulus of elasticity vs. the volume percent of WC in Co from 0 to 100 vol% using both upper- and lower-bound expressions to form a performance envelope into which the material will fall. Please do this using plotting software, not by hand.

Use Eq. 1-62 for the isostrain (upper bound) case and Eq. 1-66 for the isostress (lower bound) case.

$$E_{cP} = E_f V_f + E_m V_m$$

$$E_{c\perp} = \frac{E_f E_m}{V_f E_m + (1 - V_f) E_f}$$

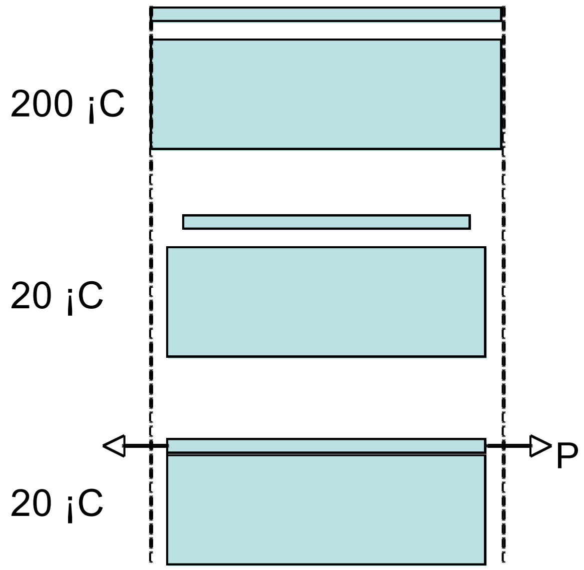


1.49 MgF_2 has the right refractive index to serve as an antireflective coating on fracture-resistant spinel ceramic windows (see problem above). Assume that the MgF_2 can be deposited as a polycrystalline thin film on thick spinel. MgF_2 mechanical properties are listed below.

	G	E	ν	CTE
MgF_2	54.5 GPa	138.5 GPa	0.27	10 x 10^{-6} °C^{-1}

(a) If a thin coating of MgF_2 is deposited on a thick polycrystalline spinel substrate at a temperature of 200 °C and then the film and substrate are cooled to 20 °C, what is the *stress state* of the thin film? Use words like equal/unequal, uni/bi/triaxial, and tension/compression. Name the state and provide a supporting sketch.

According to the data provided in an earlier problem, the spinel CTE is 7 x 10^{-6} °C^{-1}*whereas the* MgF_2 *CTE is larger, at* 10 x 10^{-6} °C^{-1}. T*he film is therefore in a state of equal biaxial tension induced by its desire to shrink more upon cooling than the substrate. The sketches* *below show cross sections of the film and substrate at 200°C (same dimensions, stress free), at 20°C if detached (films shrinks more than substrate), and at 20°C is attached (film under tension). Because the expansion occurs in all directions within the plane of the film, the stress is equal radially. There is no constraint in the direction normal to the film surface so the stress state is biaxial.*



(b) What are the thermal *strains* induced in the MgF_2 film under the conditions from part 'a'? Please give numerical answers for directions X, Y, and Z, where Z is the direction normal to the film surface.

$$\varepsilon_{th,\,biaxial} = \Delta\alpha\Delta T = (10\times10^{-6}\ °C^{-1} - 7\times10^{-6}\ °C^{-1})(200-20\ °C)=5.4\times10^{-4}$$

$$\varepsilon_{normal} = 2\left(-\nu\varepsilon_{th,\,biaxial}\right) = 2\left(-0.27\right)\left(5.4\times10^{-4}\right) = -2.92\times10^{-4}$$

(c) What is the thermal *stress* induced? Please give numerical answers for directions X, Y, and Z.

$$\sigma_{th,\,biaxial} = \left(\frac{E}{1-\nu}\right)\varepsilon_{th,\,biaxial} = \left(\frac{138.5\ GPa}{1-0.27}\right)\left(5.4\times10^{-4}\right) = 0.102\ GPa = 102\ MPa$$

$$\sigma_{th,\,normal} = 0$$

(d) If the MgF_2 were replaced by a fluoropolymer antireflective coating (like PTFE) deposited at the same temperature, would you expect the thermal *strain* in the film to be larger or smaller? Why?

Polymers like PTFE typically have much larger CTE values than ceramic materials, so one might expect the thermal strain in the film to be much larger.

Design

1.50 A solar panel is to be mounted at the top of a cylindrical post that is rigidly attached to the ground at its bottom, and that is protected from extreme bending by four guy wires strung from the top of the post to the ground. The post will be made of recycled polyethylene terephthalate (PET), which has an elastic modulus of approximately 3.5 GPa and a Poisson's ratio of 0.43. If the solar panel weighs 14.8 kg and the post must be 8 m tall to lift the panel above surrounding obstacles, what is the minimum post radius needed to avoid failure by buckling? Is this post diameter actually likely to be a safe design choice? Based only on the required post radius, what is your opinion about the choice of PET for this application?

Given rigid attachment of the post at the bottom and the presence of the guy wires arranged radially around it, it would be reasonable to assume fixed-pinned boundary conditions. The appropriate equation that describes this condition is

$$P_{cr,\,fixed-pinned} = \left(\frac{L}{\sqrt{2}}\right)^{-2} \pi^2 EI = \frac{2\pi^2 EI}{L^2}$$

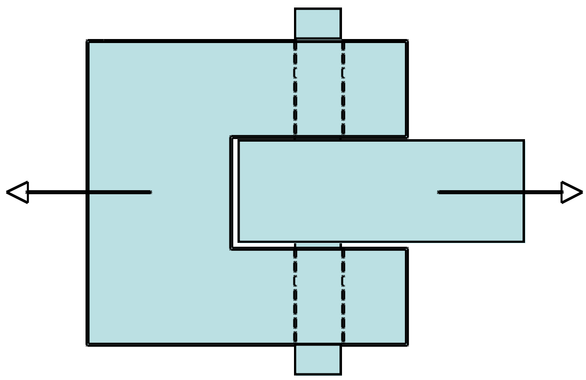
and for a circular column $I = \pi d^4/64$. Putting these together and solving for the diameter gives

$$d = \left(\frac{64 L^2 P_{cr}}{2\pi^3 E}\right)^{\frac{1}{4}} = \left(\frac{64\left(8\ m\right)^2\left(14.8 kgf\right)\left(9.807 N/kgf\right)}{2\pi^3\left(3.5\times 10^9 Pa\right)}\right)^{\frac{1}{4}} = \left(2.739\times 10^{-6} m^4\right)^{\frac{1}{4}} \approx 0.04\,m$$

$$radius = d/2 = 0.02\,m = 20\,cm$$

This would not be a safe radius to use because it has no safety factor to cover uncertainties and deviations from ideal loading. It would be better to add a safety factor of 2-3x. In this case, the radius would have to be 40-60 cm. Even without the safety factor, the radius is very large (more like a tree trunk than a slender column). Perhaps a stiffer material would be a better choice for this particular load and height requirement.

1.51 You are in charge of designing a new fixture for a "universal testing machine" that will attach a tensile specimen to the machine using a clevis — a U-shaped piece with holes drilled through the two arms — and a cylindrical pin that passes through the clevis and the specimen. If the maximum load exerted by the machine is 30 kN and the pin is to be made of some sort of steel, what is the minimum pin diameter needed to ensure that the shear stress in the pin does not exceed 600 MPa? Assume that the steel has similar elastic properties to pure Fe.



Recall that the definition of shear stress is $\tau=P/A$, where P is the force and A is the area over which the force is distributed (in this case, the cross sectional area of the pin). The minimum pin diameter is therefore

$$d_{\min} = 2\left(\frac{P}{\tau_{\max}\pi}\right)^{\frac{1}{2}} = 2\left(\frac{30\times10^{3}N}{\left(600\times10^{6}\ Pa\right)\pi}\right)^{\frac{1}{2}} = 7.98\times10^{-3}N \approx 8\,mm$$

1.52 A 20-cm-outer diameter pipe is used to carry a pressure of 1200 psi without yielding. Assuming a safety factor of 3x, compute:
(a) the lightest
(b) and the least expensive pipe per unit length
based on the following two possible material choices.

	Copper C71500	PVC
E	150	3.25
σ_{ys} (MPa)	540	43
ρ (g/cm^3)	8.94	1.45
Cost (US$/kg)	27.00	1.75

First assume that both pipes will meet the thin-wall criterion in order to determine the minimum wall thickness based on the hoop stress (i.e., the maximum stress).

$$\sigma_{hoop} = \frac{PR}{t}$$

Considering the safety factor gives a wall thickness of

$$t = 3\left(\frac{PR}{\sigma_{hoop}}\right) = 3\left(\frac{(1200\,psi)(6895\,Pa/psi)(0.020\,m)}{\sigma_{hoop}}\right) = \frac{4.9644\times10^{5}\,\frac{N}{m}}{\sigma_{ys}}$$

$$t_{Cu} = \frac{4.9644\times10^{5}\,\frac{N}{m}}{540\times10^{6}\,\frac{N}{m^{2}}} = 0.92\times10^{-3}m \approx 1mm$$

$$t_{PVC} = \frac{4.9644\times10^{5}\,\frac{N}{m}}{43\times10^{6}\,\frac{N}{m^{2}}} = 11.5\times10^{-3}m = 11.5mm$$

Now determine the mass per unit length of each pipe.

$$mass\,/\,unit\ length = (Solid\ \ Area)(Density) = \pi\left(r_{outer}^{2} - r_{inner}^{2}\right)\rho$$

$$Cu \ = \pi\left((10\,cm)_{outer}^{2} - (10-0.1cm)_{inner}^{2}\right)\left(8.94\,g/cm^{3}\right) = 55.89\,g/cm$$

$$PVC = \pi\left((10\,cm)_{outer}^{2} - (10-1.15cm)_{inner}^{2}\right)\left(1.45\,g/cm^{3}\right) = 98.75\,g/cm$$

The copper alloy pipe is approximately half the weight of the equivalent PVC pipe.

Finally, the cost per meter of pipe is

$$cost\ /\ unit\ length = (cost\ /\ unit\ mass)(mass\ /\ unit\ length)$$
$$Cu \quad = (\$27.00\ /\ kg)(1kg\ /\ 1000g)(55.89\,g\ /\ cm)(100\,cm\ /\ m) = \$150.90\ /\ m$$
$$PVC = (\$1.75\ /\ kg)(1kg\ /\ 1000g)(98.75\,g\ /\ cm)(100\,cm\ /\ m) = \$17.28\ /\ m$$

Here the PVC clearly wins, with a cost of approximately 11% that of the copper alloy.

1.53 A particular cylindrical rod will be subjected to axial cyclic compressive loads. It is designed to fit snugly through a hole in a separate plate, but it must not exert excessive pressure on the surrounding material while under load or a fatigue crack may develop in the plate. The diameter of the rod (and the hole) is 10 mm. The maximum compressive load the rod will experience is 24 kN. If the rod either yields plastically or increases in diameter by more than 0.008 mm, the design will not meet the specifications. Which of the four alloys listed below will satisfy these requirements at the lowest cost?

	E (GPa)	ν	σ_{ys} (MPa)	σ_{ts} (MPa)	US$/kg	ρ (g/cm^3)
1020 alloy steel, normalized	207	0.30	340	440	1.35	7.85
304 stainless steel, cold worked	193	0.30	510	865	8.50	8.00
Al 6061-T6	69	0.33	275	310	7.75	2.70
Ti-6Al-4V, solution & aged	114	0.34	1100	1170	125.00	4.43

First check the yield criterion. Assume that yielding in compression is the same as in tension (not a bad assumption for metals and metal alloys).

$$\sigma = \frac{P}{A_0} = \frac{P}{\pi\left(\frac{d}{2}\right)^2} = \frac{-24000N}{\pi\left(\frac{0.01m}{2}\right)^2} = -305.6\ MPa$$

This removes the Al 6061 alloy from further consideration. The 1020 alloy is close to yielding, but without a requirement of significant safety factor it is OK. Next check the diameter change criterion.

$$\Delta d = d_0\varepsilon_{radial} = -\nu d_0\varepsilon_{axial} = -\nu d_0\left(\frac{\sigma}{E}\right)$$

Finally, check the cost per unit length (all the rods will be the same size).

$$cost\ /\ unit\ length = (area)(density)(cost\ /\ mass)(conversion\ factors)$$

The results are as follows:

	Δd (mm)	US$/cm	Criteria
1020 alloy steel, normalized	0.004	1.35	Passes, best choice
304 stainless steel, cold worked	0.005	8.50	Passes, but expensive
Al 6061-T6	0.015	7.75	Fails by yielding
Ti-6Al-4V, solution & aged	0.009	125.00	Fails by excessive diameter change

1.54 A 6061-T4 aluminum alloy is to be used to make a thin-walled cylindrical canister in which high pressure chemical reactions will be performed. The design calls for a diameter of 50 cm, a length of 80 cm and a maximum operating pressure of 50 MPa. Assume a safety factor of four is required (i.e. the maximum stress can never exceed one quarter of the alloy's yield strength).

(a) What wall thickness is required to ensure safe operation?
(b) Is this wall thickness a maximum or a minimum? Explain.
(c) How do your answers change if the cylinder is made twice as long?

The maximum stress is the hoop stress, which for a cylindrical pressure vessel is

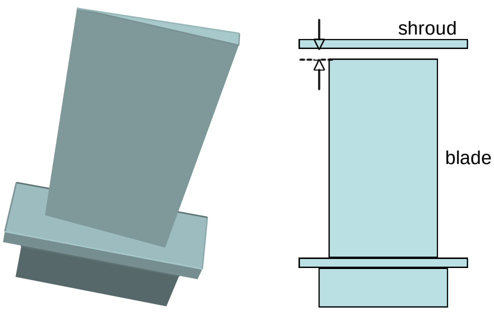
$$\sigma_{hoop} = \frac{PR}{t}$$

Considering the safety factor gives a wall thickness of

$$t = 4\left(\frac{PR}{\sigma_{hoop}}\right) = 4\left(\frac{PR}{\sigma_{ys}}\right) = 4\left(\frac{(50\,MPa)(0.025\,m)}{145\,MPa}\right) = 0.0345\,m = 3.45\,cm$$

This must be the minimum thickness, because a thinner wall would experience a higher stress and would therefore fail to meet the stress criterion. The answer will not change if the vessel is longer because length does not make any difference to the stress state (as long as it remains a cylinder).

1.55 Imagine that you are designing a single crystal turbine blade for use in a jet engine. It will experience large tensile loads from the centripetal forces that exist during use. Minimizing the axial strain will allow for tighter gap tolerances between the turbine blade tips and the surrounding shroud; this leads to greater engine efficiency. You are restricted to using a Ni-based superalloy.



(a) Without performing any calculations, determine which orientation (<100>, <111> or <110>) you would choose along the tensile axis of the blade in order to minimize the strain during use? Why?
(b) Justify your choice by calculating the Young's modulus for each orientation and then calculating the corresponding strain at maximum load. For this problem, assume that the Ni-based superalloy in question has the same elastic behavior as pure Ni. Also assume that the blade experiences a maximum load of 10,000 lbs-force, and that the behavior is elastic. Consider only a simple uniaxial tensile load. Approximate the turbine blade airfoil cross-section as an isosceles triangle 5mm at its base by 50 mm tall. The blade length is 150 mm.

(c) Calculate the thermal strain imposed on a turbine blade after it has been heated from 25°C to the engine operating temperature of 1100 °C. Assume a coefficient of thermal expansion of $13.5 \times 10^{-6}\ C^{-1}$ for this particular superalloy.
(d) Calculate the best-case minimum gap size for a cold (25°C) engine with no turbine rotation such that the blade will just barely touch the surrounding shroud material when the engine is operating at full rotation and maximum temperature.

(a) From the discussion in Chapter 1, the stiffest direction for FCC metals is along the <111> axis. The strain with therefore be the smallest in this orientation for a given load.
(b) First, calculate the modulus in each direction using the direction cosines, as per Eq. 1-55:

$$\frac{1}{E} = S_{11} - 2[(S_{11} - S_{12}) - \tfrac{1}{2} S_{44}](l_1^2 l_2^2 + l_2^2 l_3^2 + l_1^2 l_3^2)$$

Doing so, we find the following:

	E(100) GPa	E(110) GPa	E(111) GPa
Ni	137.0	232.6	303.0

It can immediately be seen that the stiffest orientation is the <111>, as expected. From this result, the strain for each orientation (hkl) is given by

$$\varepsilon = \frac{\sigma}{E} = \frac{\left(\frac{P}{A_0}\right)}{E} = \frac{(10\ kip)(4.448 \times 10^3 N / kip)}{0.5(0.005 m)(0.050 m)(E_{hkl})} = \frac{355.84 \times 10^6 Pa}{E_{hkl}}$$

	ε(100)	ε (110)	ε (111)
Ni	0.0026	0.0015	0.0012

(c) The thermal strain is given by Eq. 1-88

$$\varepsilon_{th} = \alpha(T_{final} - T_{initial}) = (13.5 \times 10^6\ °C^{-1})(1100 - 25°C) = 0.0145$$

(d) The total strain is simply the sum of the thermal and mechanical strains. The best case scenario includes the <111> orientation, for which

$$\varepsilon_{total} = \varepsilon_{mechanical} + \varepsilon_{th} = 0.0012 + 0.0145 = 0.0157$$

The minimum gap size prior to operation of the engine is equal to the expected change in blade length given by

$$\Delta L = L_0 \varepsilon_{total} = (150\ mm)(0.0157) = 2.355\ mm$$

Extend

1.56 Write a 1-2 page review of auxetic materials. Assume that you are writing a supplementary article for an introductory engineering text. Be sure to (1) define the term “auxetic material” and (2) explain what is unusual about the mechanical behavior of this class of materials. Include (3) a picture (sketch, diagram, or photograph) of an auxetic material. Also (4) describe at least two products that could (or do) benefit from the auxetic behavior. Provide full references for all of your information.

Key points:

- *An auxetic material is one that has a negative Poisson's ratio, so it expands laterally when a tensile strain is imposed axially.*
- *Not the same phenomenon as the medical term auxetic that has to do with cell expansion and division.*
- *From the Greek auxētikos = increasing*
- *Examples of auxetic materials include "reentrant" foams, certain microporous polymers, and zeolites*
- *Potential applications include filters and shock absorbing materials.*

1.57 Select two thermoplastic materials from among those listed in Section 1.3.3.1. Using any resources available to you, determine a typical glass transition temperature, degree of crystallinity, and a common use for each of the polymer materials you selected. How does the use reflect the T_g value and the degree of crystallinity for each material?

Example:

- *Thermoplastics polystyrene (PS) and high density polyethylene (HDPE) have T_g values of approximately 100°C and -90 °C, respectively.*
- *PS is amorphous. As T_g>> room temperature for PS, it tends to be rigid and fairly stiff under ambient use conditions.*
- *PS is used to make plastic cutlery and Petri dishes, two applications for which reasonable stiffness is necessary for proper function.*
- *HDPE is partially crystalline. As T_g<< room temperature for HDPE, it tends to be fairly compliant under ambient use conditions. However, because of its partial crystallinity there is a relatively small drop in stiffness associated with being above T_g, so it is much stiffer than typical elastomers (for example).*
- *HDPE is used to make milk bottles and laundry detergent bottles, for which some flexibility is desirable but extreme flexibility would not be.*

1.58 Write a 1-2 page review of the *structure* and *elastic behavior* of natural highly-elastic materials. Assume that you are writing a supplementary article for an introductory engineering text. Choose two or more materials for comparison: dragline spider silk, non-dragline spider silk, collagen, elastin, mussel byssal threads, and resilin. In your review, be sure to (1) identify the natural use for each of the materials you selected, and (2) explain how the particular properties of the materials match their intended uses in nature. Mention (3) approximately how much of the behavior is purely elastic (instantaneous recovery with no energy loss) and how much is viscoelastic (time-dependent recovery with some energy loss). Include (4) a picture (sketch, diagram, or photograph) or a plot that adds to the reader's understanding of the topic. Strength is also interesting and certainly worth mentioning, but is not the main focus of this paper. If you can find a case in which there has been an attempt to synthesize the material(s) for engineering purposes it would add much to this short article. Provide full references for all of your information.

Answers will vary widely.

1.59 Search published science and engineering literature to find an example of an engineered material used for bone replacement (partial or total). How well does the elastic behavior of the material match that of natural bone? Provide elastic property data from the source, a brief explanation of the potential advantages of this particular material, and a full reference for the source.

Answers will vary widely.

1.60 Search published science and engineering literature to find an example of a microelectromechanical device in which thermal mismatch strain is used to generate motion and/or force. Provide a figure from the source, a brief explanation of the device purpose and design, and a full reference for the source.

Answers will vary widely.

CHAPTER 2

Review

2.1 Is a *dislocation* a physical item or substance? If not what is it?

No, it is not a substance or feature made of matter that can be handled or isolated. A dislocation is a line of disruption in the crystalline arrangement of atoms. It is somewhat analogous to a crack insofar as it is a defect in a material, but it not actually composed of matter.

2.2 Why are dislocations necessary for explaining the plasticity typically seen for crystalline materials?

The theoretical stress needed for plastic deformation is much higher (usually by orders of magnitude) than the plastic deformation stress actually measured in common materials. The only way that the critical stress can be so low is if the atomic bonds associated with a slip plane are broken and reformed sequentially rather than all at once.

2.3 Identify two techniques for observing dislocations, and describe at least one strength and one weakness of each technique.

Etch pits: relatively easy to create without expensive, elaborate equipment; can only show the dislocation arrangement on a single plane, not the subsurface dislocation arrangement.

Transmissions electron microscopy: high resolution images capable of depicting complicated dislocation arrangements; requires significant specimen preparation and very thin specimens that limit the observable volume.

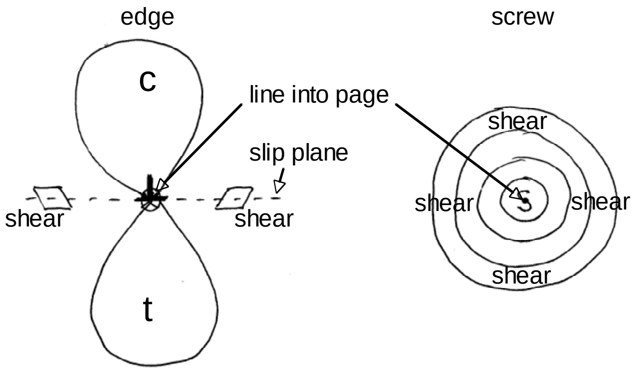
2.4 Rank the relative Peierls force in different materials and material classes and briefly explain why, in each case, this is the case.

Larger for ceramics than for metals because of the strong, directional nature of ceramic bonds. Within the metals class, higher for BCC metals than for FCC metals because BCC metals do not have "smooth" close packed slip planes like FCC metals do. Peierls stress is not particularly relevant for polymer materials because dislocations do not exist (at least in the same sense) as in metals and ceramics.

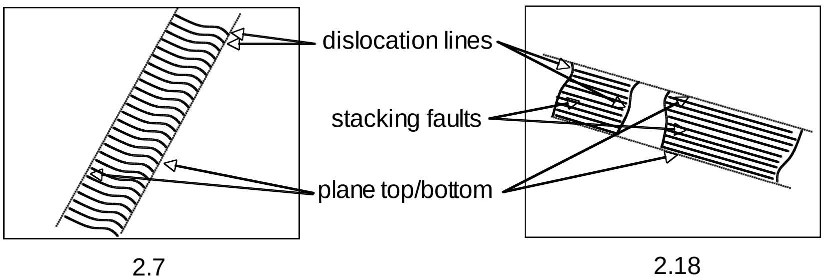
2.5 Which can cross-slip, an edge dislocation, a screw dislocation, or a mixed dislocation? Why?

Only pure screw dislocations can cross slip. The Burgers vector and the line of the dislocation are parallel to each other, so there is no unique slip plane on which the pure *screw dislocation is defined. Edge dislocation and partial dislocations all have non-parallel Burgers vectors and dislocation lines, so unique slip planes are defined that contain both vectors.*

2.6 Sketch an edge dislocation and a screw dislocation as if you are looking directly along the dislocation line in each case. Clearly mark the line direction and the slip plane (if there is a unique slip plane). Indicate on your sketches where you will find regions of hydrostatic tension, hydrostatic compression, and pure shear stress surrounding the dislocation lines.



2.7 Sketch a representative portion of the TEM images in Figures 2.7 and 2.18, including only the dislocation lines. Indicate on your sketches which features are dislocation lines, which are stacking faults, and which are the top and bottom edges of the slip planes.



2.8 Identify the crystal structure in the faulted region of an FCC crystal. Why is this the case?

*The usual close-packed plane stacking sequence for FCC is ABCABCABC, but when a stacking stack is introduced it shifts some of the planes to create ABC**ABAB**CA, which has the ABAB stacking characteristic of HCP order.*

2.9 When an FCC material has high stacking fault energy, do you expect widely-spaced or closely-spaced leading and trailing partial dislocations? Do you expect wavy or planar glide? Briefly explain both trends.

Stacking fault energy has units of energy/unit area. High stacking fault energy means that it takes a lot of energy to create additional stacking fault area, i.e., to spread the partial dislocations farther apart. So, a high stacking fault FCC material is likely to have ***closely-spaced*** *leading and trailing partial dislocations to minimize the total energy. Because they are closely-spaced, it is relatively easy to push them together to form a complete dislocation. If the leading partial encounters a barrier such as a precipitate particle, the trailing partial can be forced to join the leading partial as they press up against the barrier. When combined,* *their new Burgers vector may allow cross-slip of a portion of the complete dislocation line, so a segment will leave the original slip plane. This out of plane segment may continue to exist as the dislocation bypasses the barrier and eventually emerges from a free surface. The trace on the free surface will match the non-planar nature of the dislocation line, and will appear **wavy**.*

2.10 Consider the following face-centered-cubic dislocation reaction:

$$\frac{a}{2}[110] \rightarrow \frac{a}{6}[21\bar{1}] + \frac{a}{6}[121]$$

(a) Prove that the reaction will occur.

Following the example in the chapter

$$\frac{a^2}{4}(1 + 1 + 0) > \frac{a^2}{36}[4 + 1 + 1] + \frac{a^2}{36}[1 + 4 + 1]$$

$$\frac{a^2}{2} > \frac{a^2}{3}$$

(b) What kind of dislocations are the $(a/6)\langle 121\rangle$?

Shockley partial dislocations

(c) What kind of crystal imperfection results from this dislocation reaction?

A stacking fault between the leading and trailing partials.

(d) What determines the distance of separation of the $(a/6)[21\bar{1}]$ and the $(a/6)[121]$ dislocations?

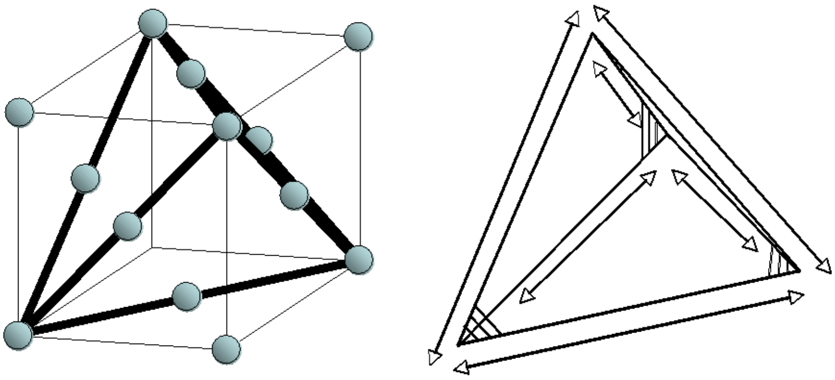
A balance between the excess energy associated with the stacking fault and the strain energy associated with overlapping the stress fields of the partials. Stacking fault energy minimization promotes small faults and thus small dislocation separations, whereas strain energy minimization promotes minimal overlap and thus large dislocation separations.

2.11 List which main slip systems are active in FCC, BCC, and HCP metals, and explain why those particular planes/directions are favored.

In the standard format of {plane}<direction>:
FCC {111}<110>
BCC {110}<111>, {112}<111>, {123}<111>
HCP {0001}<1120> or {1100}<1120>
In each case, the plane is the most closely packed available (and therefore the "smoothest"), and the direction is the most closely packed direction within the chosen plane (and therefore Gb^2 is minimized).

2.12 Sketch a 3D FCC unit cell and indicate where all 12 FCC slip systems can be found.

Four {111}-type planes and three <110>-type directions within each plane.



2.13 What does it mean to be an *independent slip system*?

An independent slip system is a combination of plane and direction that allows a shape change via slip that cannot be created using any combination of other slip systems.

2.14 What is the effect of resolved normal stress on the yield behavior of crystalline metals and ceramics?

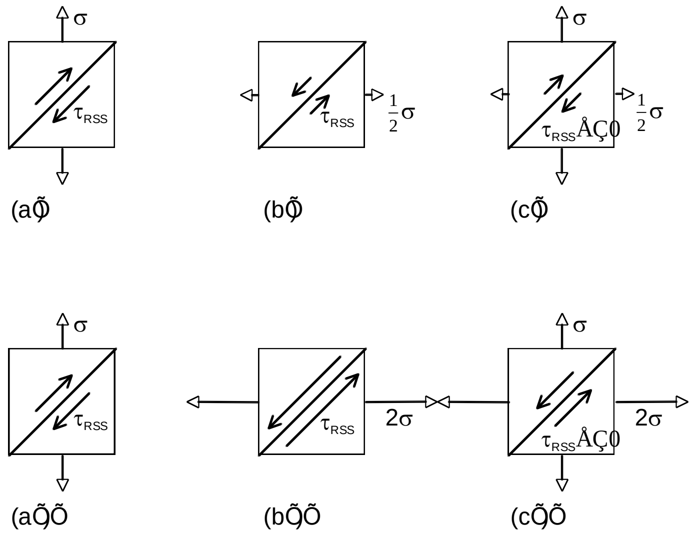
The yielding of metals and ceramics by dislocation motion is determined by the resolved shear stress on the slip plane, so the resolved normal stress plays no role.

2.15 What is the role of the *Taylor factor*?

The Taylor factor connect the concepts of the Schmid factor and a Critical Resolved Shear Stress on a particular slip plane to the behavior of polycrystalline materials that have many grain orientations, and therefore many different Schmid factors creating different resolved shear stresses in each grain. It allows the use of a single CRSS value for predicting the onset of yielding for a polycrystalline material.

2.16 Reproduce Figure 2.23 twice, first adjusting it so that it accurately depicts the case in which the horizontal stress is half that of the vertical stress, and second so that the horizontal stress is twice that of the vertical stress. Use arrow length to indicate relative stress magnitude.

Both cases will result in non-zero resolved shear stresses on the chosen slip plane even under biaxial loading, but the sign and magnitude of the shears in the two cases are not identical.



2.17 Which predicts the lower yield strength for most combinations of applied stress, the Tresca or the von Mises yield criterion? Under what stress conditions are the predictions equal?

The Tresca yield criterion has a hexagonal failure envelope that just fits inside the oval failure envelope of the von Mises yield criterion. For all stress combinations other than pure uniaxial loading or equal biaxial loading, the Tresca criterion therefore predicts a lower yield strength for a given material.

2.18 Identify the trend between stacking fault energy and strain hardening coefficient, and then use it to predict which is likely to work harden more strongly: pure copper or pure nickel, the latter of which has a stacking fault energy of approximately 240 mJ/m^2.

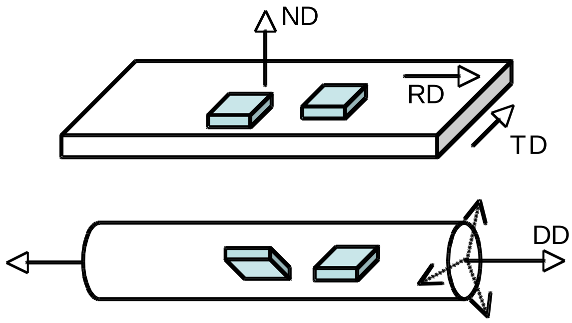
Low stacking fault energy is associated with relatively easy pinning of dislocations, so it is also associated with high strain hardening (a large strain hardening coefficient). This can be seen in the comparison between stainless steel and pure iron, for instance. Pure copper has a stacking fault energy of approximately 90 mJ/m^2 (see Table 2.5) and a strain hardening coefficient in the range of 0.3-0.35 (see Table 2.8). As pure nickel has a stacking fault energy of approximately 240 mJ/m^2, larger than that of copper, we might expect the strain hardening coefficient to be smaller.

2.19 What are the critical differences between strain hardening and geometric hardening?

Strain hardening is the result of increasing dislocation density and increasing dislocation-dislocation interactions that leads to greater resistance to plastic deformation. It is not dependent on crystal orientation. Geometric hardening is also associated with an increasing resistance to plastic deformation, but in this case the source is a decreasing resolved shear stress on the active slip system. This decreasing trend is due to the rotation of the grain that contains the dislocations as the macroscopic shape of the specimen is changed. Geometric hardening can happen without a change in dislocation density or dislocation-dislocation interaction.

2.20 Describe how a wire texture is different than rolling texture, and sketch an example of each with arrows indicating the directions of preferred orientation.

Rolling texture has three preferred directions: the Rolling Direction (RD), the Transverse Direction (TD), and the Normal Direction (ND). A wire texture has only one preferred direction (along the wire drawing axis, DD). Grain orientation is random in the radial direction of the wire. This sketch shows the two cases, with representative "grains" illustrating the alignment with ND, RD, and TD for the rolling texture but only with DD for the wire texture.



2.21 After a dislocation has passed through a crystal, thereby causing plastic deformation, what does the inside of the crystal look like? Contrast this with the appearance of the interior of a crystal that has deformed by twinning.

A dislocation breaks bonds and reforms them as it passes through the crystal, leaving behind the same atomic order that existed before the dislocation passed through. A twin reorients the crystal so that the twinned region is visible after the twinning process is complete. A metallographic specimen etched to bring out grain contrast can show evidence that deformation twinning took place at some time in the past, but it does not show such clear evidence of prior dislocation activity.

2.22 Under what conditions is twinning favored in BCC and/or FCC crystals?

At high strain rates and low temperatures — conditions under which plasticity by dislocation motion is difficult.

2.23 What is the basic molecular mechanism for polymer plasticity, and how does it differ from that of ductile crystalline metals?

Polymer plasticity occurs by sliding of the polymer chains past each other. The chains must move as units, so polymer plasticity does not occur by dislocation motion.

2.24 What specific aspects of polymer molecule structure (e.g., side group size, shape, polarity, and location) favor chain sliding?

Easy chain sliding is favored by smaller, simpler side groups with low polarity. Polymer chains with side groups randomly arranged along the length (atactic) have difficulty packing into crystalline arrangements, so they are easier to slide past one another than those that have tighter packing (isotactic or syndiotactic).

2.25 How does the structure of a crystalline polymer differ from that of a crystalline metal? What are the implications of this difference for plasticity in both classes of material?

A crystalline polymer structure is formed by folding the polymer chains back and forth against themselves to create segments of high alignment (crystals). The backbone chain bonds remain intact, however, so the atoms in the chain cannot act independently. Metals arrange in crystalline order by packing individual atoms. As a result, chain sliding in a polymer must involve the cooperative behavior of many atoms whereas metal plasticity can involve discrete events of atom repositioning via dislocation motion.

2.26 What are the two micro-scale plasticity mechanisms active in amorphous polymers? Are they likely to occur simultaneously? Explain.

The two plasticity mechanisms active in amorphous polymers are crazing and shear banding. They are usually competing processes, so they are unlikely to occur simultaneously.

2.27 Explain the role of *crazing* in determining the extent of maximum plastic deformation for some polymers. Be sure to include both tensile and compressive loading in your answer.

Crazing is a plasticity mechanism that acts in tension (but not compression), and that may create microcracks within the polymer. These microcracks can serve as nucleation sites for macrocracks that lead to failure and can therefore limit the ductility in tension. In compression no microcracks can form, and so the polymer is more likely to remain intact when subjected to large amounts of plastic strain.

2.28 What typically happens to the strength level of a polymer that has undergone *cold drawing*? Why?

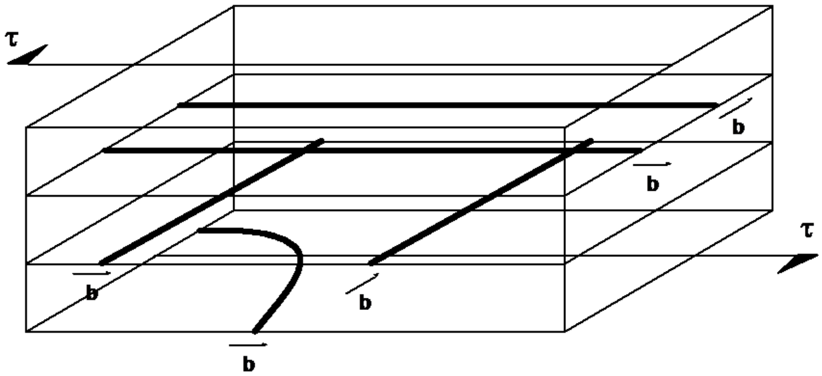
Cold drawing involves alignment of the polymer chains with the tensile (drawing) axis. When this occurs, the strong C-C bonds along the chain backbone bear most of the load. The strength level in the drawing direction therefore increases as a result of the cold drawing process.

2.29 What is the typical effect of resolved normal stress on the yield behavior of polymeric materials? Is this same as for crystalline metals and ceramics?

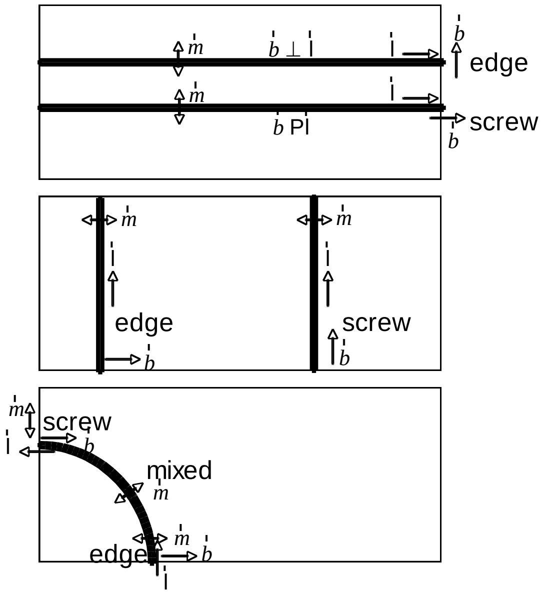
Unlike metals and ceramics, polymers can exhibit large asymmetries in yield stress when loaded in tension vs. compression. A tensile resolved stress on a shearing plane opens up space and allows for greater chain mobility. Conversely, a compressive resolved stress crowds the chains together, making them more difficult to slide past one another. Compression therefore tends to increase the yield strength of a polymer. Metals and ceramics behavior similarly in tension and compression with regard to plastic yielding by slip. Differences may well occur when twinning is the prevailing deformation mechanism..

Practice

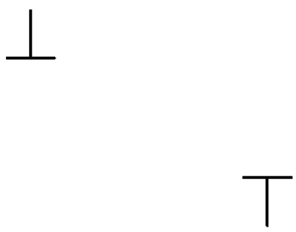
2.30 The dislocations shown below (on three separate slip planes) represent different characters. Assume that no negative edge or left-hand screw dislocations are included.



(a) Sketch this diagram, then clearly identify the character of each dislocation by writing a label nearby. How do you know each type?
(b) How does each dislocation behave under the applied shear shown on the diagram? Sketch the dislocation lines and indicate the direction of motion, if any, on three separate projections of the slip planes (i.e., as seen from above).



2.31 Two edge dislocations of opposite sign are found in a material separated by several planes of atoms as shown below.



Please provide a helpful sketch and an explanation along with the answer for each of the following questions:
(a) Without changing slip planes, will they spontaneously line up one under the other?

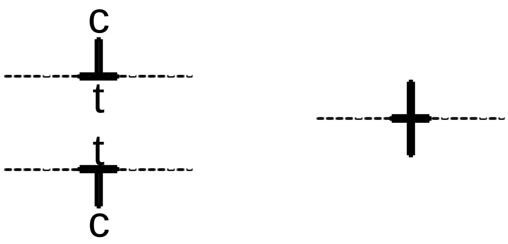
No, because it would cause the tensile strain zone of each to overlap with the other, greatly raising the energy level. (Shown on the left of the diagram below.)

(b) Under what circumstances could they move to the same slip plane?

If one or both can climb.

(c) If they did so, what would tend to happen once they were on the same plane?

They would tend to move toward each other, eventually annihilating. (The diagram on the right shows the "extra half planes" merging to form a single plane, at which point the dislocations would cease to exist.)



2.32 For austenitic stainless steel, Cu, and Al (all FCC metals):

(a) Calculate the actual magnitudes of the full and partial dislocations, assuming that the lattice parameters are 0.365 nm, 0.362 nm, and 0.405 nm, respectively.

The magnitudes are: $\left|b_{[110]}\right| = \sqrt{\frac{a^2}{2^2}[1^2 + 1^2 + 0^2]}$ *and* $\left|b_{[211]}\right| = \sqrt{\frac{a^2}{6^2}[2^2 + 1^2 + 1^2]}$

(b) Calculate the equilibrium partial dislocation separation distance d for all three materials.

The equilibrium spacing is $d = \frac{G(\mathbf{b}_2\mathbf{b}_3)}{2^1\gamma}$ *where b_2 and b_3 are the magnitudes of the leading and trailing partial dislocations.*

(c) Put the numbers from part (b) in context by comparing them to the atomic size (diameter) and lattice parameter for each material.

	lattice a (nm)	[110] b full (nm)	[112] b partial (nm)	G (Pa)	SFE (J/m^2)	spacing d (nm)	diameter D (nm)
stainless	0.365	0.258	0.149	8.16E+10	1.00E-02	28.84	0.248
Cu	0.362	0.256	0.148	4.83E+10	9.00E-02	1.87	0.256
Al	0.405	0.286	0.165	2.61E+10	2.50E-01	0.45	0.286

The atom diameters are all similar in size to one another. The atom diameter is essentially the same as the full dislocation magnitude, as you might expect if you are going to shift a plane of atoms by one atomic position. The equilibrium partial dislocation spacing is less than two atom diameters (~1.6) for high stacking fault energy Al, but more than one hundred (~116.3) atom diameters for low stacking fault energy stainless steel. A huge difference in stacking fault size relative to atom size!

(d) In which of the three material(s) is wavy glide very likely to be observed?

Wavy glide is observed when cross-slip is easy, which is the case for high stacking fault energy materials. From the list above, Al certainly qualifies.

2.33 A cube of material is loaded triaxially resulting in the following stresses at the point of plastic yielding: $\sigma_x = 140$ MPa, $\sigma_y = 20$ MPa, and $\sigma_z = 35$ MPa.

(a) What is the *shear strength* of the material according to the Tresca yield criterion?

According to the Tresca criterion, yielding occurs when the maximum shear stress is

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2} \geq \tau_{CRSS}$$

So in this case, $\tau_{CRSS} = \frac{140\,MPa - 20\,MPa}{2} = 60\,MPa$

(b) If the stress in direction Z at failure were 70 MPa instead, how does this change your result? Explain.

It makes no difference because directions X and Y are still the maximum and minimum, respectively.

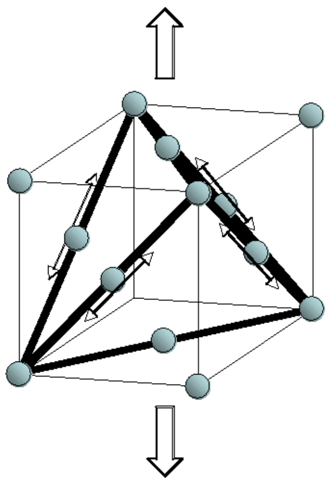
2.34 A single-crystal rod of FCC nickel is oriented with the [001] direction parallel to the rod axis.

(a) Identify the type of slip system involved in the plastic flow of nickel.

{111}<110>

(b) How many such slip systems are in a position to be activated at the same time when the load is applied parallel to this crystallographic direction?

8 systems (4 planes, 2 directions per plane), as shown in this diagram:



(c) What is the Schmid factor for this slip system? (The angles between the {100} and {110} and {100} and {111} planes are 45 and 54.7°, respectively.)

$$\phi = 54.7° \quad \lambda = 45°$$

$$\cos\phi\cos\lambda = \cos(54.7)\cos(45) = 0.4086$$

2.35 From the work of D. C. Jillson, *Trans. AIME* **188,** 1129 (1950), the following data were taken relating to the deformation of zinc single crystals.

ϕ	λ	*F* (newtons)
83.5	18	203.1
70.5	29	77.1
60	30.5	51.7
50	40	45.1
29	62.5	54.9
13	78	109.0
4	86	318.5

The crystals have a normal cross-sectional area of 122×10^{-6} m^2.
ϕ = angle between loading axis and normal to slip plane
λ = angle between loading axis and slip direction
F = force acting on crystal when yielding begins

(a) Identify the slip system for this material.

For Zn, c/a>1.633 so basal slip is favored: $\{0001\}\langle 11\bar{2}0\rangle$

(b) Calculate the resolved shear τ_{RSS} and normal σ_n stresses acting on the slip plane when yielding begins.

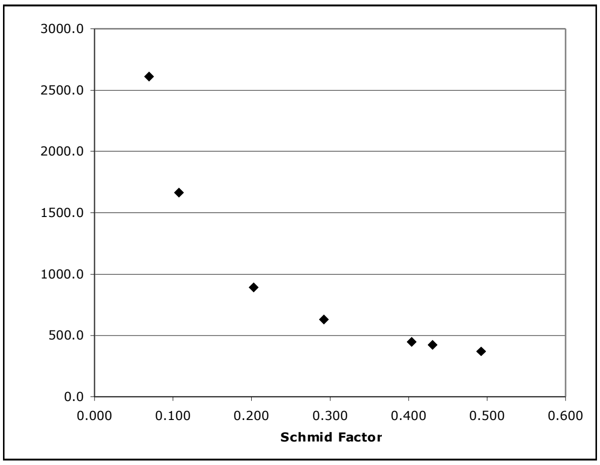
$$\tau_{RSS} = \frac{P}{A_0}\cos\phi\cos\lambda \quad \text{and} \quad \sigma_n = \frac{P}{A_0}\cos^2\phi$$

ϕ	λ	*F* (N)	Schmid	τ RSS (kPa)	σ n (kPa)
83.5	18	203.1	**0.108**	**179.2**	**21.3**
70.5	29	77.1	**0.292**	**184.5**	**70.4**
60	30.5	51.7	**0.431**	**182.6**	**105.9**
50	40	45.1	**0.492**	**182.0**	**152.7**
29	62.5	54.9	**0.404**	**181.7**	**344.2**
13	78	109	**0.203**	**181.0**	**848.2**
4	86	318.5	**0.070**	**181.7**	**2598.0**

(c) From your calculations, does τ_{RSS} or σ_n control yielding?

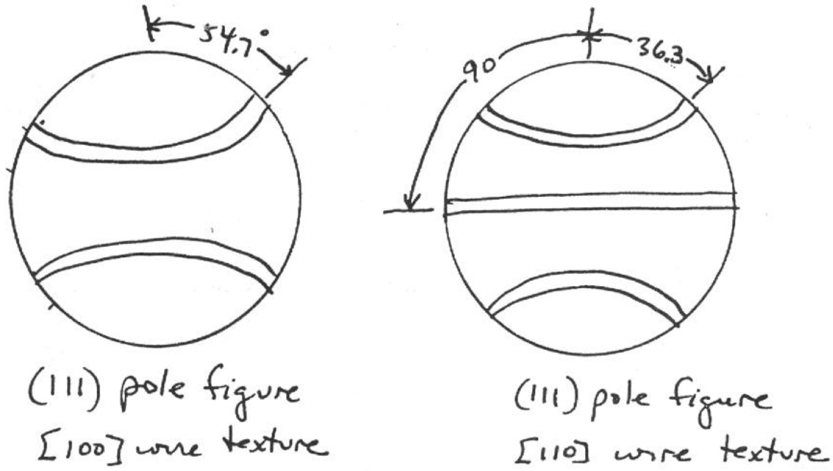
The resolved shear stress values are all nearly identical at the point of yielding, whereas the resolved normal stresses are very inconsistent. The shear stress must therefore control yielding.

(d) Plot the Schmid factor versus the normal stress P/A_0 acting on the rod. At what Schmid factor value are these experimentally-measured yield loads at a minimum? Does this make sense?



The minimum load required for yielding corresponds to a Schmid factor of 0.5. This makes sense because the critical resolved shear stress would be at a maximum when the slip plane and direction are both at 45° to the loading axis.

2.36 Draw the (111) pole figure for the [100] wire texture in silver and for the [110] wire texture in iron wires.



2.37 A low-carbon steel alloy was loaded in tension until just after yielding took place. A few Lüders bands were visible on the surface. The bar can either be reloaded (a) immediately, (b) after a brief and moderate temperature aging treatment, or (c) after several weeks without any exposure to elevated temperature. In each of the three cases, how is the yield strength of the reloaded bar likely to compare to that of the original test?

The presence of Lüders bands implies that this material exhibited an upper and lower yield point. If only a few bands are visible, it is likely that the material is in the lower yield point condition at the time of unloading. If reloaded immediately without any further treatment, the measured yield strength will match the lower yield strength of the original test. If reloaded after a "brief and moderate temperature aging treatment", i.e. a strain aging treatment, it is likely that some (or all) of the C and the N impurities in solution will have diffused to the dislocations so the measured yield strength will approach or match the upper yield strength of the original test. If reloaded after several weeks, sufficient time for diffusion will probably *have occurred and again the measured yield strength will approach or match the upper yield strength of the original test.*

2.38 The tensile strength for cold-rolled magnesium alloy AZ31B plate is approximately 160 MPa for specimens tested either parallel or perpendicular to the rolling direction. When similarly oriented specimens are compressed, the yield strength is only 90 MPa. Why? (*Hint:* Consider the possible deformation mechanisms available in the magnesium alloy and any crystallographic texture that might exist in the wrought plate.)

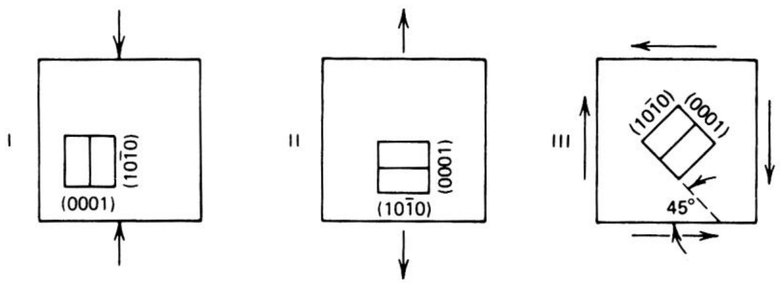
The cold-rolled plate of AZ31B develops a sheet texture $\{0001\} < 2\bar{1}\bar{1}0 >$. *As such, tensile loading in the plane of the sheet will lead to slip since the basal plane is stretched. In compression, however, the plate would deform by twinning and slip with the yield strength being lowered.*

2.39 An HCP alloy, known as Hertzalloy 200, has a *c/a* ratio of 1.600.

(a) Identify the most probable slip system for this material.

Prism slip in the close-packed direction : $\{10\bar{1}0\} < 11\bar{2}0 >$

(b) For each of the following diagrams, determine whether slip will occur and whether twinning will occur (consider only $\{10\bar{1}2\}$ twinning). Briefly justify your answers.



The c/a ratio of 1.6 is less than $\sqrt{3}$, *so the material behaves like beryllium with regard to twinning.*

Orientation	*Slip?*	*Twinning?*
I	*No, no shears on prism planes because all are parallel to the loading axis*	*No, basal plane perpendicular to tensile loading direction*
II	*Yes, shears exist on certain prism planes (although drawing is 2D, the pictured prism planes are at an angle out of the paper with respect to the loading axis)*	*No, basal plane parallel to tensile loading direction*
III	*Yes, shears exist on certain prism planes*	*No, effectively loaded like case II*

2.40 Assume that the yield behavior of PMMA is well-described by the pressure-modified von Mises yield criterion, that yielding occurs under pure shear loading at $\sigma_1 = -\sigma_2 = 60$ MPa, and that yielding occurs under pure tension loading at $\sigma_1 = 94.2$ MPa. Predict the stress needed to cause yielding (a) in uniaxial compression along direction 1 or 2, (b) under equal biaxial tension, and (c) under equal biaxial compression. Finally, plot these yield conditions in a fashion similar to that of Fig. 2.73.

The pressure-modified von Mises yield criterion is $\tau_{VM} = \tau_{oct0} + \mu_{VM} P$, *where* τ_{VM} *is the critical shear stress for yielding as a function of pressure,* μ_{VM} *is a pressure coefficient, and P is the mean pressure. The quantity* τ_{oct0} *is the critical octahedral shear stress determined in the absence of hydrostatic pressure (i.e., in pure shear). We need to know the quantities* τ_{oct0} *and* μ_{VM} *in order to solve for the yield conditions under all possible loading scenarios. First, use the result from the pure shear loading experiment:*

$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}$ *so for this case in which* $\sigma_1 = -\sigma_2 = 60\ MPa$

$$\tau_{oct0} = \frac{1}{3}\sqrt{(\sigma_1+\sigma_1)^2+(-\sigma_1)^2+(\sigma_1)^2} = \frac{1}{3}\sqrt{(2\sigma_1)^2+2(\sigma_1)^2} = \frac{1}{3}\sqrt{6\sigma_1^2} = \frac{\sqrt{6}}{3}\sigma_1 \approx 49\,MPa$$

Now, use the pure tension loading experiment to solve for μ_{VM}*:*

$\tau_{VM} = \frac{\sqrt{2}}{3}\sigma_Y$ *for uniaxial loading,* $\tau_{VM} = \tau_{oct0} + \mu_{VM}P$ *for polymers,*

and $P = -\frac{1}{3}(\sigma_1+\sigma_2+\sigma_3) = \frac{-\sigma_1}{3} = \frac{-94.2\,MPa}{3} = -31.3\,MPa$

therefore $\mu_{VM} = \frac{1}{P}\left(\frac{\sqrt{2}}{3}\sigma_Y - \tau_{oct0}\right) = \frac{1}{-31.3\,MPa}\left(\frac{\sqrt{2}}{3}(94.2\,MPa) - 49\,MPa\right) \approx 0.15$

Now, with all the pieces in place, yielding occurs when

$\tau_{oct} \geq \tau_{oct0} + \mu_{VM}P$, *so the critical condition is*

$$\frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2} = 49\,MPa + (0.15)\left(-\tfrac{1}{3}(\sigma_1+\sigma_2+\sigma_3)\right)$$

which, for uniaxial loading, is

$$\frac{1}{3}\sqrt{(\sigma_1)^2+(\sigma_1)^2} = 49\,MPa + (0.15)\left(-\tfrac{1}{3}\sigma_1\right)$$
$$\frac{\sqrt{2}}{3}|\sigma_1| = 49\,MPa - (0.05)\sigma_1$$
$$\frac{\sqrt{2}}{3}|\sigma_1| + (0.05)\sigma_1 = 49\,MPa$$

There are two values of σ_1 *that will satisfy this expression: one for unaxial tension, and one for unixial compression. Using a numerical solver (such as the Excel Goal Seek or Solver functions) returns these values as +94 MPa (as expected) and -116 MPa.*

For equal biaxial tension ($\sigma_1 = \sigma_2$) we once again invoke the general yield criterion for this material

$$\frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2} = 49\ MPa + (0.15)\left(-\tfrac{1}{3}(\sigma_1+\sigma_2+\sigma_3)\right)$$ to give

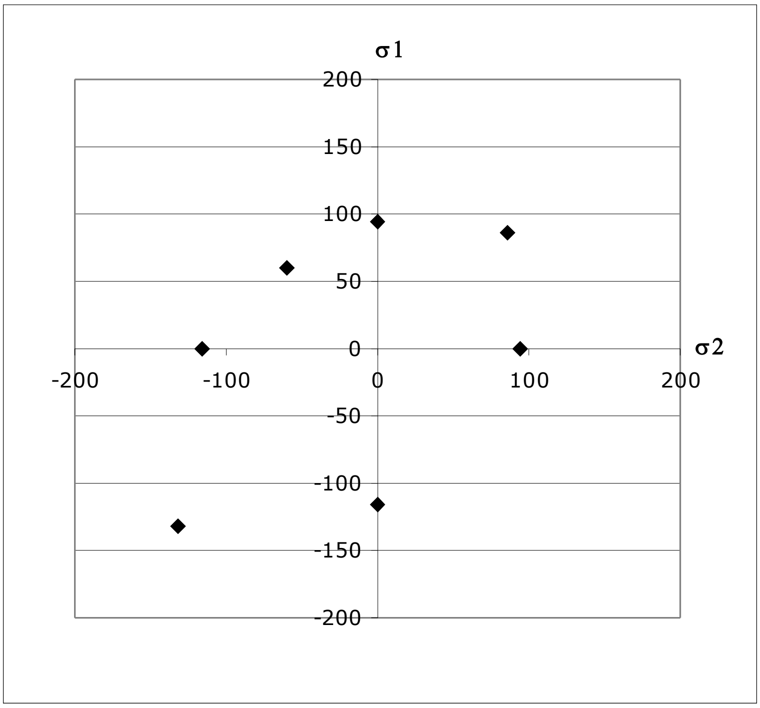
$$\frac{1}{3}\sqrt{(\sigma_1-\sigma_1)^2+(\sigma_1-0)^2+(\sigma_1-0)^2} = 49\ MPa - (0.05)(\sigma_1+\sigma_1+0)$$

$$\frac{1}{3}\sqrt{(\sigma_1)^2+(\sigma_1)^2} = 49\ MPa - (0.05)(2\sigma_1)$$

$$\frac{\sqrt{2}}{3}|\sigma_1| = 49\ MPa - (0.10)\sigma_1$$

$$\frac{\sqrt{2}}{3}|\sigma_1| + (0.10)\sigma_1 = 49\ MPa$$

Once again using a numerical solver, we get $\sigma_1 = \sigma_2 = +86$ MPa and $\sigma_1 = \sigma_2 = -132$ MPa. Finally, plotting the points gives the following graph.



Design

2.41 The design of a metallic component is undergoing a change such that the stress state will go from pure uniaxial tension to biaxial loading, with the secondary load applied at a 90° angle to the primary load, and with the secondary stress always at 20% of the primary stress but of opposite sign. The material in question is a plate of rolled 304 stainless steel in the annealed state, and the original uniaxial tensile design stress was 50% of the yield strength (to achieve a safety factor of 2x). The change in loading conditions will require a corresponding design change to the maximum allowed tensile stress in order to continue to meet the 2x safety factor. What fraction of the original tensile stress is still allowed under the new, biaxial loading condition?

The goal of the safety factor in this case is to prevent yielding. As this is a metallic component, either the standard Tresca or von Mises yield criterion could be used to evaluate this situation. For simplicity, we'll use the Tresca criterion here. The critical shear stress under uniaxial loading is half the yield strength, but we don't have to worry about the actual value as long as the shear stress doesn't change. The maximum shear stress is given by

$\tau_{\max} = \dfrac{\sigma_{\max} - \sigma_{\min}}{2}$ *which for the unaxial case is simply* $\tau_{\max} = \dfrac{\sigma_1}{2}$*. We must maintain the maximum resolved shear stress at this same level to maintain the 2x safety factor, which is equivalent to the numerator remaining the same:*

$$\sigma_1 = \sigma_{1,reduced} - \sigma_2 = \sigma_{1,reduced} + 0.2\sigma_{1,reduced} = 1.2\sigma_{1,reduced}$$
$$\sigma_{1,reduced} = 0.83\sigma_1$$

so the allowed tensile stress in the new design is 5/6 that of the original.

2.42 An unplasticized PVC component is intended to be loaded under uniaxial tension. After premature failure of a prototype component, a cross section observed under crossed polarizing lenses reveals that there are shear bands extending from the surfaces into the material, but none in the interior. What, if anything, does that tell you about the actual loading of the component in service that could be used to guide changes to the design?

Shear bands will appear where the local stress has exceeded the yield strength. Because the shear bands only appear only at the surfaces, it implies that the surfaces experienced higher strength than the interior when the component was under load. This would be the case if the loading had a bending component and was not pure tension, as intended.

2.43 You have been asked to use a finite element computer model to predict the yield condition for a PMMA component under a complex loading scenario. Among the many yield criteria that are likely to be available in the finite element software package, which would probably be the best choice for this case? Why?

PMMA is likely to have a pressure-sensitive yield response, so one of the pressure-sensitive yield criteria would probably be a good choice. In particular, Quinson et al. (1997) found that PMMA was best described by the modified von Mises criterion, so this may be the best option.

Extend

2.44 Acquire a journal paper that uses Neumann Bands as evidence in a failure analysis. Summarize the article, clearly identify the role that the discovery of Neumann Bands played in the failure analysis, and provide a formal reference for the paper.

Answers will vary widely.

2.45 Find five examples of products made of plasticized PVC and five made of unplasticized PVC. How does the choice of plasticized vs. unplasticized PVC match the engineering requirements of the products in each category?

Answers will vary widely.

CHAPTER 3

Review

3.1 List and briefly define the five main strengthening mechanisms in metals.

Strain (work) hardening: Once a reasonable dislocation density is established, increasing dislocation density leads to increased dislocation-dislocation interactions that impede dislocation motion.

Boundary strengthening: Internal interfaces such as grain boundaries or lamellar phase boundaries act as barriers to dislocation motion. Closely spaced boundaries provide the greatest strength.

Solid solution strengthening: Solute atoms alter their local environment by changing the stiffness and distorting the lattice planes. Certain combinations of dislocation type and altered environment can impede dislocation motion.

Precipitation hardening: Precipitates (second phases) grown as a result of thermodynamic driving forces within the material act as barriers to dislocation motion. Thermal processing can alter the particle size, shape, distribution, volume fraction, and nature of the phase boundary.

Dispersion strengthening: Hard second phase particles mixed into the material act as barriers to dislocation motion. Not sensitive to thermal processing.

3.2 Calculate the approximate total dislocation line length (or range of lengths) expected in a cubic cm of a very highly cold-worked metal.

According to Section 3.2, a high dislocation density is $\geq 10^{10}/cm^2$. *In Section 3.3, dislocation densities as high as* 10^{11} *to* 10^{13} *dislocations/cm*^{2} *are reported for highly cold-worked metals. Converting to line length/unit volume by multiplying each density number by cm/cm gives the following range:*

$10^{10}/cm^2 - 10^{13}/cm^2 = 10^{10}\ cm/cm^3 - 10^{13}\ cm/cm^3 = 10^{8}\ m/cm^3 - 10^{11}\ m/cm^3$

So on the highest end of the range, over a billion meters of dislocation line length are present in a single cubic cm of material. Needless to say, that's a lot.

3.3 Does the strength of a metal always increase as dislocation density increases? Explain.

No, if the initial dislocation density is very low (approaching zero) then plasticity is limited mostly by the lack of dislocations available to move. In this regime, increasing dislocation density provides more mobile dislocations and the strength declines. Once a significant *number of dislocation-dislocation interactions begin to take place, further increases in dislocation density result in strength increases. In practical terms, the latter situation is the most common so strain hardening is more common than strain softening.*

3.4 In Fig. 3.2, must it be true that the dislocation segments at points C and C' are screw dislocations? And must it be true that they are of opposite sign?

The segments do not have to be screw dislocations unless the original pinned dislocation segment was an edge dislocation. The line sense of the segment marked C will be perpendicular to that of the original dislocation, so if the original was a pure edge then C will be pure screw. If the original had screw character, the segment at C would be of edge character. At C' the line sense will also be perpendicular to the original segment, but pointing in the opposite direction as at point C (the line sense orientation with respect to the line must be continuous around the loop). As a result, whatever its character is, it must be of the opposite sign as the segment at C so annihilation is ensured.

3.5 If a series of dislocations produced by a single Frank-Read source encounter an impassible barrier, a *back stress* is created. What is the reason for this back stress, and why is it very effective for dislocations produced by a single F-R source?

The back stress is a direct result of the lattice distortion and associated stresses generated around a dislocation. A Frank-Read source will generate identical dislocations on a single glide plane, so when they are pushed up against one another their identical stress fields overlap with maximum repulsive energy.

3.6 How do dislocation junctions contribute to strengthening?

Junctions can turn mobile dislocations into sessile dislocations, preventing them from contributing to further plastic deformation.

3.7 Define *cell wall* and explain its role in strengthening.

A cell wall is a cluster of dislocations that have grouped together in a low energy configuration, surrounding relatively dislocation-free zones. A clear zone surrounded by a dense cluster of dislocations is called a cell, so the cluster serves as the cell wall. As cell walls develop and cell size shrinks, there is a high hardening rate. Once the cells are fully developed, however, the hardening rate declines. The more clearly defined the cells are, the clearer are their interiors. This is the case for high stacking fault energy materials, in which the likelihood of cross slip processes needed to organize the cell walls is high. Clear interiors allow for easy dislocation motion inside the cell, so the absolute hardening is lower than for materials that form less distinct cell structures (i.e., those with low stacking fault energy).

3.8 Define *hot work* and *cold work* in the context of dislocations and microstructure stability, then give an estimate of the temperature (or fraction of the melting point) that usually marks the transition between the two.

From Section 3.3, "When mechanical deformation at a given temperature causes the microstructure to recrystallize spontaneously, the material is said to have been hot worked. If *the microstructure were stable at that temperature, the metal experienced cold working. The temperature at which metals undergo hot working varies widely from one alloy to another but is generally found to occur at about one-third the absolute melting temperature. Accordingly, lead is hot worked at room temperature, while tungsten may be cold worked at 1500°C." Hot working allows recrystallization (growth of dislocation-free grains as a way of reducing defect energy), a process that eliminates many dislocations and softens the material. Cold working does not allow recrystallization so the dislocation density is high and strength is high.*

3.9 What data would you collect, and what axes would you use to make a linear plot for determining the grain size dependence of yield strength according to the Hall-Petch relationship?

Collect yield strength as a function of grain size (d) or lamellar spacing (whatever defines the smallest distance between potential barriers). Plot yield strength vs. $d^{-0.5}$ *to give a straight line. The slope will be* K_{HP}*, the Hall-Petch coefficient, and the intercept will give the strength of the material in the absence of boundary strengthening.*

3.10 State two different possible functions of a grain boundary that are often invoked to explain Hall-Petch behavior.

Usually the boundary is thought to act as a barrier to dislocation motion, but it can also be a source of dislocations. More closely spaced barriers allow only limited dislocation motion and severely restrict the generation of new dislocations from sources via back stresses. A high density of dislocation sources would provide a path for rapid increases in dislocation density during straining, so dislocation motion would also be highly restricted after a certain amount of cold work. In both cases the end result is essentially the same — many immobile dislocations.

3.11 Explain which has a larger effect on solid solution strengthening — symmetrical or asymmetrical point defects — and identify which specific defects lead to symmetrical or asymmetrical stress fields. List at least one example of an engineering material in which this factor comes into play.

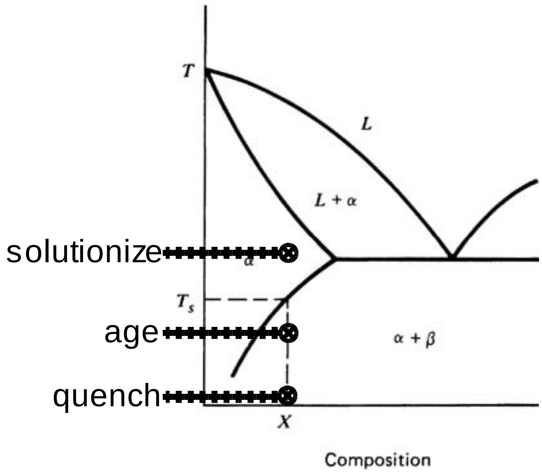
Asymmetrical point defects provide the greater strengthening because they generate both hydrostatic and shear strains in the surrounding lattice. Edge dislocations can interact with either type of lattice strain, but screw dislocations are only affected by shear distortions. Asymmetrical point defects therefore can interact with a larger fraction of dislocations present than symmetrical defects that generate only hydrostatic strains in the surrounding lattice. Asymmetrical point defects play a major role in the success of steel (C in Fe) as a structural material.

3.12 What do the formation of *Lüders bands* and *dynamic strain aging* have in common, and how are they different?

Both are evidence of inhomogeneous plasticity, and both stem from dislocations breaking away from solute atmospheres. The biggest difference is that the inhomogeneous deformation associated with Lüders bands spreads across the material until dislocations nearly everywhere have broken away from their solute atmospheres. During dynamic strain aging *the solute atmospheres are able to diffuse quickly enough to catch up to many of the free dislocations as they move, repinning them. Instead of a nearly-constant lower yield stress, dynamic strain aging leads to a series of serrated drops in strength superimposed on a rising strength level associated with strain hardening.*

3.13 Reproduce the binary phase diagram depicted in Fig. 3.20. For the composition shown at X, mark on the diagram approximate temperatures used for the three main thermal process steps used in precipitation hardening: solution treatment, quenching, and aging.

The solution treatment must be above the solvus line so only a single phase is thermodynamically favored, the quench temperature must be well below the solvus line to encourage precipitate nucleation (high driving force) but inhibit precipitate growth (slow diffusion), and the aging temperature must be high enough to encourage precipitate growth without dissolving the precipitates back into solution.



3.14 What is another name for *Ostwald ripening*, and what effect does the process have on mechanical strength?

Ostwald ripening is another name for coarsening. In this process, the volume fraction of precipitates remains unchanged but precipitate size and spacing both increase. Small, closely spaced precipitates are generally best for strengthening, so Ostwald ripens leads to overaging and loss of strength.

3.15 Describe the conditions that favor cutting of a particle vs. looping of a dislocation around a particle.

If the misfit strain between the precipitate and the lattice is small and the interface is coherent, it is possible for a dislocation to cut through a precipitate particle. If it is also the case that the spacing between particles is very small, looping around the particles may be too difficult and cutting may dominate. If the misfit strain is large, the interface incoherent, or the average particle separation is above a certain critical value, dislocations are either unable to cut through the precipitate or the stress needed to cause cutting is simply greater than that to cause looping; in these case the dislocations loop around individual particles instead of cutting through them.

3.16 Describe the trends between strength change and increasing particle spacing or particle size.

In the initial stages of aging, increasing particle size is mainly associated with increasing particle volume fraction, and also with increasing lattice distortion. Changes in particle spacing are relatively minor. Strength therefore increases. In the later stages of aging, a constant volume fraction of precipitate particles is achieved and any further increases in particle size come at the expense of particle spacing. There is also the possibility that coherency will be lost. The net effect is to transition to a looping mechanism and to decrease strength.

3.17 Why are Ni_3Al (γ') precipitates particularly effective at strengthening nickel-based superalloys?

First, the Ni_3Al (γ') precipitate is an ordered phase. When a dislocation cuts through the particle it creates an antiphase boundary that is a high energy interface. The antiphase boundary energy is sufficiently high that its formation provides a significant energy barrier to cutting. A second contribution is the very high volume fraction of precipitates in γ- γ' superalloys, as seen in Fig. 3.23b. Thus there is a large particle volume blocking dislocation motion no matter what the direction of motion within the material.

3.18 Compare and contrast precipitation strengthening and dispersion strengthening.

Both mechanisms strengthen by using hard particles to block dislocation motion. However, precipitates are second phases that are grown as a result of thermodynamic driving forces within the material. Thermal processing can alter the particle size, shape, distribution, volume fraction, and nature of the phase boundary. In the case of dispersion strengthening, hard second phase particles made of compounds (typically oxides) that are foreign to the parent material are mixed into the parent material in the solid state, often through a powder metallurgy process. Because these particles are foreign to the parent material (i.e., not soluble) and are usually very thermodynamically stable phases, they are not sensitive to thermal processing. This complicates their fabrication, but makes them relatively insensitive to elevated temperatures during use.

3.19 Describe a method by which "ODS" alloys are produced and explain why it is not possible to cast these alloys using conventional techniques.

Oxide dispersion strengthened alloys consist of a metallic parent phase and hard second phase particles made of oxide compounds. The metal atom within the oxide is generally different from the parent metal, so the particles are foreign to the parent material and are generally not soluble. As a result, they cannot be formed by melting and solidifying a group of elements. Instead, the alloys may be formed through a mechanical alloying process in which the pre-formed particles are mixed with a powder of the parent material until a good dispersion is achieved. The resulting powder particles can then be consolidated into useful shapes for further fabrication or direct use.

3.20 What is the difference between an *intrinsic* and an *extrinsic* strengthening mechanism, and on which do metal matrix composites depend?

An intrinsic strengthening mechanism is one that fundamentally changes the ease of the plasticity processes occurring within the material, making them more difficult to create increased strength. An extrinsic strengthening mechanism is one that provides strength by shifting much of the load to a second material that is mixed into the main material, and that is stronger than the main material. Metal matrix composites are a good example of extrinsic strengthening because the behavior of the parent metal is not significantly altered by the presence of reinforcing particles, but the overall composite strength is greater than that of the metal alone.

3.21 What are three advantages of fiber-metal laminates over conventional metals for certain aircraft applications?

The specific weight (mass per unit volume) of an Al/fiber composite FML is lower than that of solid aluminum, which improves aircraft fuel economy. Also, the aluminum alloy outer layers provide impact resistance and damage detectability as compared to conventional fiber composites. Furthermore, cracks that may initiate in the aluminum alloy surface layers are arrested when the crack front encounters the fiber composite layer. This greatly extends overall fatigue lifetime for components fabricated with the FML material. (There are also benefits, not described in the chapter, associated with the layers acting as moisture barriers. This improves corrosion resistance.)

3.22 What aspect of thermoset polymers can be controlled to increase strength?

The degree of cross-linking can be used to control strength. Greater cross-linking leads to greater strength. For example, the development of strong cross-links is the main process underlying the curing of two-part epoxies that have sufficient strength to be used for structural adhesive applications.

3.23 What are two fundamental differences between orientation strengthening of metals vs. polymers? The first should address the mechanism by which strengthening is achieved, and the second the thermal stability of the high strength characteristic.

Orientation strengthening of metals is achieved when grains rotate during slip. The development of a preferred texture can put the primary slip planes at orientations with respect to the loading axis that make further slip difficult. Polymer orientation strengthening, on the other hand, occurs when the molecular chains align along the primary tensile axis. The alignment of the strong C-C covalent bonds gives these aligned fibers great strength. A metal can be annealed at high temperature, causing grain growth and loss of the texture that was responsible for the orientation strengthening. Grains do not exist in the same fashion in a polymer, so the alignment of the polymer chains is not very sensitive to elevated temperatures and the material tends to retain its strength after elevated temperature exposure (below the melt temperature, of course).

Practice

3.24 A Frank-Read source created from a {111}<110> dislocation segment in Al is observed in a transmission electron microscope. Using a straining stage, it is possible to load the specimen to watch the source in action. If the pinning points are 55 nm apart, estimate the minimum resolved shear stress necessary to cause the dislocation segment to become unstable, thereby generating a new loop.

The stress necessary to produce the looping instability is given by $\tau \approx \frac{Gb}{l}$ *so we need the shear modulus and Burgers vector for a {111}<110> dislocation in Al. From Chapter 1, G=26.1 GPa, and from Chapter 2 we can figure out that* $\left|b_{[110]}\right| = \sqrt{\frac{a^2}{2^2}[1^2 + 1^2 + 0^2]}$. *Inserting the Al lattice parameter a=0.405 nm, we therefore find b=0.286 nm. Finally,*

$$\tau \approx \frac{Gb}{l} = \frac{(26.1\,GPa)(0.286\,nm)}{55\,nm} = 0.1357\,GPa = 135.7\ MPa$$

3.25 Experimentally, it has been observed for single crystals that the critical resolved shear stress τ_{CRSS} is a function of the dislocation density ρ_D as

$$\tau_{CRSS} = \tau_0 + A\sqrt{\rho_D}$$

where τ_0 and A are constants. For copper, the critical resolved shear stress is 0.69 MPa at a dislocation density of 10^4 mm^{-2}.

(a) If it is known that the value of τ_0 for copper is 0.069 MPa, please calculate the τ_{CRSS} at a dislocation density of 10^6 mm^{-2}.

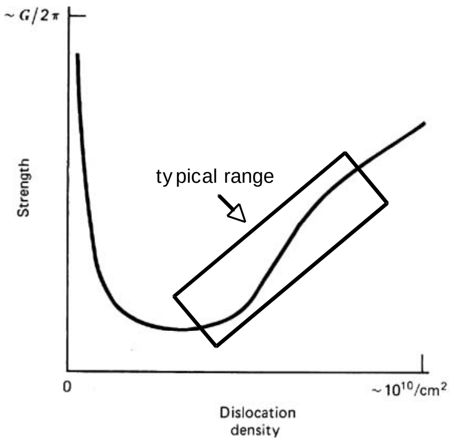
We must first determine the constant A from

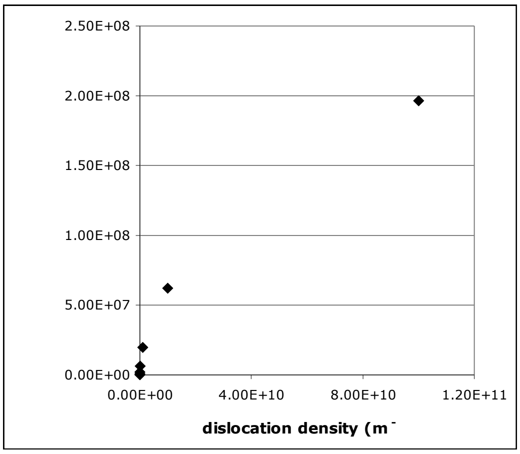
$$A = \frac{\tau_{CRSS} - \tau_0}{\sqrt{\rho}} = \frac{0.69\times 10^6\,\frac{N}{m^2} - 0.69\times 10^5\,\frac{N}{m^2}}{\sqrt{10^{10}\,m^{-2}}} = 6.21\,\frac{N}{m}$$

then for a dislocation density of 10^6 *mm*$^{-2}$

$$\tau_{CRSS} = 0.69\times 10^5\,\frac{N}{m^2} + 6.21\,\frac{N}{m}\sqrt{10^{12}\,m^{-2}} = 6.279\ MPa$$

(b) Plot τ_{CRSS} for Cu over a dislocation density range of 10^3 to 10^{10}/cm^2. Does your plot look like some or all of Fig. 3.1? What does this mean with regard to the validity of the equation?





The plot looks like the mid- to high-density part of the "typical range" depicted in Fig. 3.1. This underscores the fact that the expression given for relating the critical resolved shear stress to the dislocation density is only relevant to the "normal" dislocation density range for metals. It will not accurately model much lower densities.

3.26 When making hardness measurements, whether by nanoindentation or by conventional indentation testing, what will be the effect of making an indent very close to a preexisting indent? Why?

The act of making an indentation will strain harden the material in the surrounding zone. If another indent is placed too close to the first, it will return an artificially high hardness because of the prior work hardening.

3.27 Summarize the general effect that Stacking Fault Energy has on the ability of an FCC metal to work harden, then briefly describe *three* mechanisms by which this influence occurs. The *first* of your answers should address the interaction between dislocations and second phase particles, the *second* should address dislocation junctions, and the *third* should address cell development.

Low stacking fault energy in an FCC material is associated with widely spaced partial dislocations. This affects work hardening by (1) making it difficult to cross-slip around second phase particles, (2) creating a situation in which leading partial dislocations can create sessile junctions like stair-rods dislocations, thereby shutting down glide, and (3) by making cross-slip difficult so that a dislocation cell structure with dislocation-free zones does not develop quickly. In all three cases, the low stacking fault energy plays an important role in strengthening.

3.28 The lower yield point for a certain plain carbon steel bar is found to be 135 MPa, while a second bar of the same composition yields at 260 MPa. Metallographic analysis shows that the average grain diameter is 50 µm in the first bar and 8 µm in the second bar.

(a) Predict the grain diameter needed to cause a lower yield point of 205 MPa.

Dislocation motion in the lower yield point plateau is not affected by significant work hardening, so the change in yield strength can safely be attributed to just the difference in grain size. The Hall-Petch relation can therefore be used to determine the relationship between grain size and yield strength.

$\sigma_{ys} = \sigma_i + k_y d^{-1/2}$ so

$\sigma_{2ys} - \sigma_{1ys} = \left(\sigma_i + k_y d_2^{-1/2}\right) - \left(\sigma_i + k_y d1^{-1/2}\right) = k_y\left(d_2^{-1/2} - d_1^{-1/2}\right)$

$$\therefore \quad k_y = \frac{\sigma_{2ys} - \sigma_{1ys}}{\left(d_2^{-1/2} - d_1^{-1/2}\right)} = \frac{260\ MPa - 135\ MPa}{\left(8\ \mu m\right)^{-1/2} - \left(50\ \mu m\right)^{-1/2}} = 595\ MPa\sqrt{\mu m}$$

Although the units may look odd, they are convenient in this case. We don't know the intrinsic strength, but we don't need it because we can use the difference in the strength.

$\sigma_{2ys} - \sigma_{3ys} = k_y\left(d_2^{-1/2} - d_3^{-1/2}\right)$

$$\therefore \quad d_3 = \left[d_2^{-1/2} - \left(\frac{\sigma_{2ys} - \sigma_{3ys}}{k_y}\right)\right]^{-2} = \left[\left(8\ \mu m\right)^{-1/2} - \left(\frac{260\ MPa - 205\ MPa}{595\ MPa\sqrt{\mu m}}\right)\right]^{-2} \approx 14.5\ \mu m$$

(b) If the steel could be fabricated to form a stable grain structure of 500 nm grains, what strength would be predicted?

Following a similar procedure as in parts (a) and (b) gives 2711 MPa, or 2.711 GPa. A very high strength!

(c) Why might you expect the upper yield point to be more alike in the first two bars than the lower yield point?

The upper yield point is determined by the stress needed to break dislocations away from their solute atmospheres, so the grain size should not have a strong effect (unless the dislocation density is so high and/or the grains are so small that a back stress influences the breakaway stress).

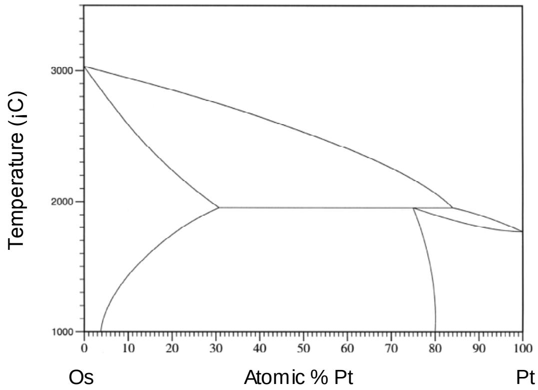
3.29 A high-carbon steel with a fully pearlitic microstructure was used to form a high-strength bolt (H.-C. Lee et al., J. Mater. Proc. Tech. 211, p. 1044, 2011). It was found that the pearlite in the bolt head had an average interlamellar spacing of 257 nm whereas the average spacing in the body of the bolt was 134 nm. Assuming that dislocation pile-up is the primary mechanism responsible for the strength of this alloy, what ratio of strength (or hardness) might be expected between the head and body of the bolt?

$\sigma_{ys} \propto k_y d^{-1/2}$

$$\frac{\sigma_{ys,head}}{\sigma_{ys,body}} = \frac{d_{head}^{-1/2}}{d_{body}^{-1/2}} = \frac{\left(257\,nm\right)_{head}^{-1/2}}{\left(134\,nm\right)_{body}^{-1/2}} = 0.72$$

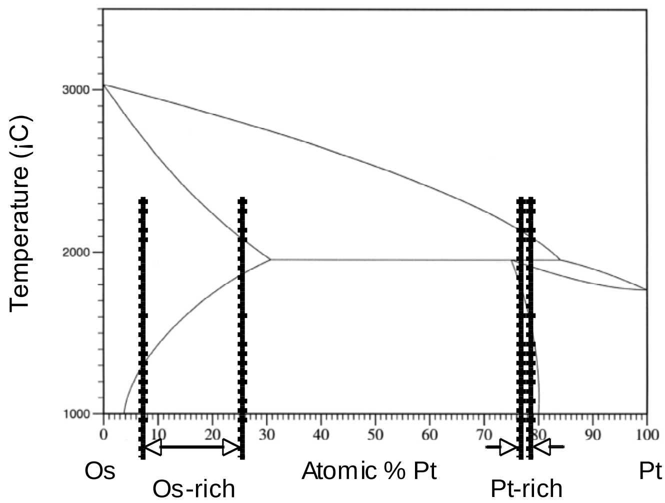
so the strength (or hardness) of the bolt head is likely to be only 72% that of the bolt body.

3.30 A sketch of the Os-Pt binary phase diagram is provided below.



(d) Reproduce the binary phase diagram and mark on it approximate candidate composition ranges for precipitation strengthened Osmium-rich and Platinum-rich Os-Pt alloys.

The ranges must reach into a solid solution (single phase region) and into the two-phase region. Realistically, there must be some room for error in the solution heat treatment temperature. Also, there must be sufficient room beneath the solvus line to allow quenching to room temperature and aging at an elevated temperature.



(e) From a processing perspective alone, would precipitation strengthening be equally practical to achieve in Osmium-rich and in Platinum-rich Os-Pt alloys? Explain your reasoning.

Both alloys are technically feasible to achieve, but composition control would be more important for the Pt-rich case because the solvus line is very steep and allows very little room for error. This would make the Os-rich alloy more forgiving with regard to processing.

(f) Assume that an Osmium-rich precipitation strengthened alloy is created. What other strengthening mechanisms are likely to be acting in the same alloy? List any critical assumptions behind the existence of each mechanism you believe is relevant.

The primary strengthening mechanism would clearly be precipitation hardening. In addition, there will be some contribution of solid solution strengthening because even once a two-phase microstructure is achieved, the matrix phase will be a stable Os-rich solid solution of about 4 *at% Pt. If the alloy is polycrystalline, as most conventional alloys are, boundary strengthening may play a role. However, the effect would only be significant if the grain size is small. Finally, the alloy could be cold-worked to add a strain hardening contribution. This is not necessary, but would be a likely option. The strain hardening exponent would have to be determined in order to assess the potential significance of this mechanism.*

3.31 The lattice parameters of Ni and Ni_3Al are 3.52 × 10^{-10} m and 3.567 × 10^{-10} m, respectively. The addition of 50 at% Cr to a Ni-Ni_3Al superalloy increases the lattice parameter of the Ni matrix to 3.525 × 10^{-10} m. Calculate the fractional change in alloy strength associated with the Cr addition, all other things being equal.

Misfit hardening goes as $\tau \propto G\varepsilon^{3/2}\left(rf\right)^{1/2}$ *so if the shear modulus, particle radius, and particle volume fraction are all unchanged by the Cr addition, we predict that*

$$\frac{\tau_{Cr}}{\tau_0} = \frac{\varepsilon_{Cr}^{3/2}}{\varepsilon_0^{3/2}}$$

In order to calculate the strength ratio we need the lattice misfits given by

$\varepsilon=(a_{ppt} - a_{lattice})/a_{lattice}$.

Without Cr: $$\varepsilon_{misfit} = \frac{3.567\times10^{-10}\,\text{m-}3.52\times10^{-10}\,\text{m}}{3.52\times10^{-10}\,\text{m}} = 0.0134 = 1.34\%$$

Without Cr: $$\varepsilon_{misfit} = \frac{3.567\times10^{-10}\,\text{m-}3.525\times10^{-10}\,\text{m}}{3.525\times10^{-10}\,\text{m}} = 0.0119 = 1.19\%$$

$$\frac{\tau_{Cr}}{\tau_0} = \frac{\left(0.0119\right)^{3/2}}{\left(0.0134\right)^{3/2}} = 0.837$$

The strength after the Cr addition is only 83.7% of the original strength. Coherency between the Ni_3Al *particles and the Ni lattice is improved with the addition of Cr, so we would expect the misfit strengthening to be reduced accordingly.*

3.32 The addition of C to Fe greatly increases the room-temperature strength of the alloy, but an equal amount of C added to Ag has little effect. Why?

Fe at room temperature has the BCC crystal structure. C atoms are very small compared to Fe atoms, so the C sits at interstitial sites in the Fe lattice. The octahedral BCC sites are asymmetric in shape, so the lattice distortion associated with the presence of the C atoms (slightly too large to fit into the site) is also asymmetric. On the other hand, Ag has the FCC crystal structure, which only has symmetric interstitial sites into which the C will fit. Asymmetric point defects provide greater strengthening because they generate both hydrostatic and shear strains in the surrounding lattice. Edge dislocations can interact with either type of lattice strain, but screw dislocations are only affected by shear distortions. Asymmetric point defects therefore can interact with a larger fraction of dislocations present than symmetrical defects that generate only hydrostatic strains in the surrounding lattice.

3.33 Some alloys use a combination of strain hardening and precipitation hardening to achieve particularly high strength levels. The usual order of strengthening is solution treatment, quenching, cold working, and finally precipitation heat treatment. Why not reverse the order of the cold working and precipitation heat treatment steps?

If the alloy is precipitation hardened prior to cold working, the high degree of initial hardness will make the cold-working step much more difficult. Also, the material may have limited ductility as a result of the high hardness, so the metal may crack during the cold-working process.

Design

3.34 Provide a reasonable explanation for the following observation: a welded component made of Al 6061-T6 alloy is routinely found to deform plastically first in the region adjacent to the weld joint despite the fact that the stress is nominally the same everywhere in the component. What solution would you propose to fix this problem, assuming that the weld joint cannot be eliminated from the design?

Welding inherently involves exposing the solid material surrounding the weld zone to high temperatures. Furthermore, cooling of the heated region after the weld pass is complete is usually rapid because there is a large thermal mass associated with the rest of the component. Alloy 6061-T6 is a precipitation hardened Al-Si-Mg alloy in an artificially aged condition (aged for peak strength). Uncontrolled heating of the adjacent material probably caused the precipitates to go back into solution, then rapid cooling either caused the Cu to stay in solution or to precipitate in an uncontrolled fashion. The solution for this problem is to put the entire component through the solution-quench-age precipitation heat treatment sequence after welding to ensure uniform properties throughout.

3.35 An aircraft fuselage design calls for a 2/1 layup of GLARE laminate (2 layers of Al and 1 layer of glass/epoxy prepreg). A study of this material (H. F. Wu, L. L. Wu, W. J. Slagter, J. L. Verolme, J. Matl. Sci. 29 (1994) 4583-4591) found that the 0.38 mm thick glass prepreg layer had an ultimate strength of 1507 MPa in the longitudinal direction (along which 70% of the glass fibers were aligned), while the 0.3 mm thick 2024-T3 aluminum layers each had an ultimate strength of 490 MPa. The density of the 2/1 layup was 2.45 g/cm^3. The density of 2024 alloy is 2.77 g/cm^3. The study also found that the laminate followed the rule of mixtures (Eq. 3-25) with regard to density and to ultimate tensile strength. It is proposed that a change from a 2/1 laminate to a 3/2 laminate for the aircraft fuselage offers the opportunity to reduce overall vehicle weight by reducing the composite density. Calculate the density of the 3/2 layup to check this assertion, then calculate the ultimate tensile strength of both laminate to ensure that there is no significant tradeoff with regard to ultimate strength. Finally, briefly discuss any other potential drawbacks that would have to be evaluated before selecting the 3/2 laminate over the 2/1 laminate.

The density of the 2024 alloy is known, but we need the density of the prepreg to calculate the overall density of the 3/2 layup. First determine the volume fraction of Al in the 2/1 layup, then use this to determine the prepreg density. With this information and the volume fraction of Al in the 3/2 layup, the density of the 3/2 layup can be determined.

Total 2/1 thickness: 2(0.3 mm)+0.38 mm = 0.98 mm, so the volume fraction of Al is 0.6/0.98=0.61 or 61%. The rule of mixtures (volume weighted average) is used to calculate the prepreg density.

$$\rho_{2/1}^{Lam} = V_f^{Al}\rho^{Al} + (1 - V_f^{Al})\rho^{pre}$$
$$2.45\ g/cm^3 = 0.61\left(2.77\ g/cm^3\right) + (1 - 0.61)\rho^{pre}$$
$$\rho^{pre} = 1.95\ g/cm^3$$

Total 3/2 thickness: 3(0.3 mm)+2(0.38 mm) = 1.66 mm, so the volume fraction of Al is 0.9/1.66=0.54 or 54%. The rule of mixtures is used to calculate the 3/2 laminate density.

$$\rho_{3/2}^{Lam} = 0.54\left(2.77\ g/cm^3\right) + (1 - 0.54)\left(1.95\ g/cm^3\right)$$
$$\rho_{3/2}^{Lam} = 2.39\ g/cm^3$$

As anticipated, the density of the 3/2 layup (2.39 g/cm^3) is lower than that of the 2/1 layup (2.45 g/cm^3) because the volume fraction of prepreg is greater in the 3/2 case.

The ultimate strength can also be predicted using the rule of mixtures and the Al volume fraction for the 3/2 layup.

$$\sigma_{2/1,ult}^{Lam} = V_f^{Al}\sigma_{ult}^{Al} + (1 - V_f^{Al})\sigma_{ult}^{pre} = 0.61\left(490\ MPa\right) + \left(1 - 0.61\right)\left(1507\ MPa\right)$$
$$\sigma_{2/1,ult}^{Lam} = 886.6\ MPa$$

$$\sigma_{3/2,ult}^{Lam} = V_f^{Al}\sigma_{ult}^{Al} + (1 - V_f^{Al})\sigma_{ult}^{pre} = 0.54\left(490\ MPa\right) + \left(1 - 0.54\right)\left(1507\ MPa\right)$$
$$\sigma_{3/2,ult}^{Lam} = 957.8\ MPa$$

We conclude that there is not only a decrease in density associated with moving to the 3/2 laminate, there is also an increase in the ultimate strength. However, the 3/2 laminate is thicker so the absolute weight of a given panel will be greater than the 2/1 layup. Also, the cost associated with the additional Al and prepreg layers will be higher. Finally, properties other than ultimate strength will undoubtedly be important (e.g., yield strength, impact resistance, fatigue resistance). These factors would all have to be considered, along with any possible design changes enabled by the greater strength and thickness, before making a decision to adopt the 3/2 laminate.

Extend

3.36 A processing technique called Equal Channel Angular Pressing (ECAP) has been used for many metals and alloys to impose severe plastic deformation, and therefore create extreme dislocation densities. However, the primary purpose of ECAP is not to create high strength by severe work hardening. What is the main reason to perform ECAP processing, and why is the technique particularly attractive for this purpose?

The main purpose of ECAP is to create severe plastic deformation that results in an extremely small final grain size, ideally sub-micrometer, in order to achieve very high strength without the need for alloying elements in significant quantities. It is very attractive because it does not *change the cross-sectional area of the material (unlike rolling) and it may be possible to scale it up for production purposes.*

3.37 Find a journal paper that describes either an experimental study or a simulation of the so-called "reverse/inverse Hall-Petch" phenomenon. To what mechanism does the paper attribute the phenomenon, and over what grain size range is it claimed to act? Provide a full reference for the paper in a standard reference format.

Answers will vary.

3.38 Look up the standard aluminum alloy heat treatment temper designations. Use the designations to justify the yield strength behavior of the following aluminum alloys listed in Table 1.2: 2024-T3 vs. 2024-T6 and 7075-T6 vs. 7075-T73.

Tables will vary somewhat. The T3 temper indicates that the alloy has been solution heat treated, cold worked, and naturally aged. The T6 tempered indicates a solution heat treatment followed by artificial aging to peak strength, and the T7 condition indicates a solution heat treatment followed by artificial aging past the peak strength (i.e., overaging). These designations match the trend in the yield strength values listed in Table 1.2. Of the two 2024 conditions, the T6 temper has the higher strength (395 MPa vs. 345 MPa). Of the two 7075 conditions, the T6 is the stronger (505 MPa vs. 415 for the T7 case). Note that the 2024 and 7075 alloys have different chemistries, which is the reason for the difference between their strengths in the T6 condition.

3.39 Why are rivets of a 2017 aluminum alloy often refrigerated until the time they are used?

Alloy 2017 is a precipitation hardened Al alloy that age hardens at room temperature. In the hardened state, the alloy is too hard to drive as a rivet (which requires significant plastic deformation to form the head of the fastener once it is in place). It must be solution heat treated and quenched immediately before driving, or it can be refrigerated for a limited time to slow the aging process.

3.40 Briefly discuss the potential benefits and the obstacles to the use of ferritic (i.e., steel-based) ODS alloys for nuclear power applications. Provide full references in a standard reference format for any papers you used to develop your understanding.

Answers will vary.

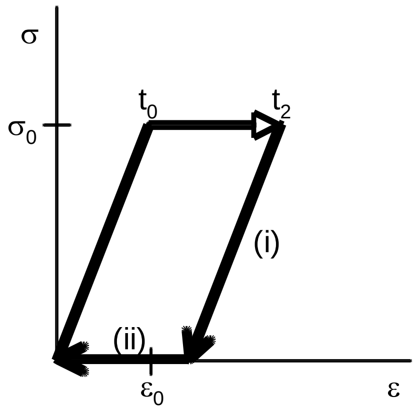
3.41 Find publications that describe the structure and properties of *Araneus* MA silk and *Araneus* viscid silk. Use what you learn about the structure of these materials to explain the differences in the stiffness, strength, and extensibility reported in Table 3.3. Provide full references in a standard reference format for any papers you used to develop your explanations.

Answers will vary.

CHAPTER 4

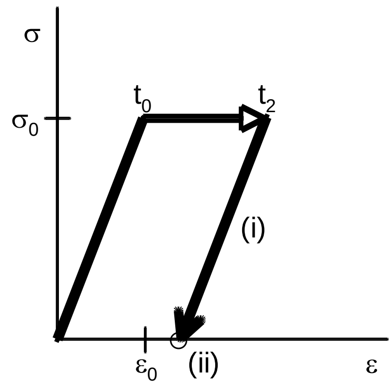
Review

4.1 Reproduce Fig. 4.1a. Imagine that the material in question is *viscoelastic*, and that very rapid elastic loading is followed by creep deformation over a long period of time designated by t_2. Add to the sketch two additional curves showing the behavior (i) during rapid unloading to zero stress, and (ii) during a long period of time ($t \gg t_2$) at zero stress after unloading.



The lines added to the figure show (i) a path that matches the loading slope during rapid unloading, and (ii) slow recovery of the viscoelastic strain. If the behavior is 100% viscoelastic, the strain should eventually reach zero when the stress is zero.

4.2 Follow the instructions for Review Question 1, but for a material that is *viscoplastic*.



The unloading path marked (i) matches the slope of the original loading curve, but because the material is 100% viscoplastic there is no recovery of the creep strain. The final (permanent) strain is given by the point marked (ii).

4.3 State at what homologous temperature creep may start to become an issue for metals and for ceramics. Then estimate the actual temperature for Al, Ti, ZrO_2, and SiC. Is the onset of creep behavior with respect to temperature sudden or gradual?

The onset of creep is a gradual process that is thermally activated. For crystalline metals, time-dependent deformation processes usually do not become significant until the homologous temperature is approximately 0.3 or higher. For Al, one might start to consider creep as a possibility above (660 °C + 273 °C)/3 = 311 K = 38 °C. Depending on alloying and prior heat treatment, however, it may be that the actual creep rate is too low for concern at this temperature. For Ti, one might start to consider creep as a possibility above (1668 °C + 273 °C)/3 = 647 K = 374 °C.

*Due to strong directional atomic bonding, ceramic materials may not experience creep or stress relaxation in any meaningful way until a homologous temperature exceeding 0.4-0.5. For zirconia this is approximately (2715 °C + 273 °C)*0.45 = 1345 K = 1072 °C. SiC does not melt per se (at atmospheric pressure it decomposes at sufficiently high temperature) but a lower limit on creep may be determined as ~(2730 °C + 273 °C)*0.45 = 1351 K = 1078 °C.*

4.4 What factors determine whether polymer behavior is predominantly elastic, viscoelastic, or viscoplastic?

Temperature, time of interest, degree of cross-linking, and degree of crystallinity are the key factors. At temperatures below T_g and for short-time loading, polymer behavior will be mostly elastic. At temperatures near or above T_g, and for long loading times, polymer strain may have substantial viscoelastic or viscoplastic components. Highly cross-linked and highly crystalline polymers have little or no viscoplastic contribution to their overall behavior at any temperature, so the time-dependent deformation that appears for $T \geq T_g$ is essentially all reversible (i.e., viscoelastic). On the other hand, amorphous polymers (or lightly crystallized polymers with a large amorphous fraction) can show a range of behavior from a nearly elastic behavior for $T<T_g$ to a nearly viscous behavior for $T>T_g$ (but still below T_m), and they show a particularly strong viscoelastic contribution when $T \approx T_g$.

4.5 Describe the standard loading conditions for a creep test. Explain why these loading conditions are chosen.

Either constant load or constant stress. As a general rule, data being generated for engineering purposes are obtained from constant load tests, while more fundamental studies involving the formulation of mathematical creep theories should involve constant stress testing so that changes in cross-sectional area can be included.

4.6 What two general competing processes control the creep rate of a metal? What is the relative strength (or rate) of the two processes during Stage I and Stage II creep behavior?

The two processes are strain hardening and softening (recovery) that occur simultaneously. The hardening rate is greater than the softening rate in Stage I, leading to a decrease in creep rate as strain increases and strength increases. When the rates of the two processes balance, Stage II behavior appears as an approximately-constant creep strain rate.

4.7 What leads to a change from Stage II to Stage III behavior? Be specific about the mechanisms involved, and what can influence the time of this transition.

At high stress and/or temperature levels, the balance between hardening and softening processes is lost, and the accelerating creep strain rate in the tertiary stage is dominated by a number of weakening metallurgical instabilities. Among these microstructural changes are *localized necking, corrosion, intercrystalline fracture, microvoid formation, precipitation of brittle second-phase particles, and dissolution of second phases that originally contributed toward strengthening of the alloy. In addition, the strain-hardened grains may recrystallize and thereby further destroy the balance between material hardening and softening processes.*

4.8 Describe under what circumstances the steady state creep rate may be a more useful parameter than rupture life, and vice versa.

Steady state creep rate is more useful when designing for long component lifetime, and when dimensional stability is critical to component performance. Rupture life is more useful when designing for limited component lifetime, and when conditions are sufficiently harsh that fracture as a result of accumulated creep damage is a strong possibility.

4.9 Describe the general trends that connect stress level and temperature with steady state creep rate and rupture life.

Increasing stress and increasing temperature are both associated with increasing steady state creep rate and with reduced rupture life.

4.10 Why do the curves in Fig. 4.4 have more than one linear segment, and what does the transition from one segment to the next indicate?

In this plot the rupture life decreases with increasing stress, as expected, but there appear to be different stress and temperature regimes that are distinct from one another. Grant and Bucklin identified different metallurgical instabilities that were operating at different stress and temperature levels, each of which is associated with a linear segment of a failure curve. The slope changes therefore indicate a change in the dominant creep failure mechanism.

4.11 What data would have to be collected to determine the stress exponent for creep? What plot axes would give a linear relationship using this data set?

Determine the steady state creep rate as a function of applied stress for several levels of stress, holding temperature constant. Plotting log (or ln) stress vs. log (or ln) strain rate will generally give a linear relationship, assuming that a single mechanism is involved.

4.12 What data are typically collected to determine the activation energy for the controlling creep mechanism?

Determine the steady state creep rate as a function of temperature, holding stress constant.

4.13 How would you know if there were different creep mechanisms acting at different temperatures?

Plotting the log of the steady state creep rate as a function of inverse absolute temperature (i.e., 1/T) should give a straight line if a single mechanism is operating. A change of slope at a particular temperature indicates a change in mechanism.

4.14 The plot in Fig. 4.5 shows significant differences in the creep rates of α- and γ-iron tested at the same temperature, 910°C. How can this difference be explained?

The creep rate is dependent on the self-diffusion rate, and the self-diffusion rate is faster in α-iron because its BCC crystal structure is less densely packed than the FCC crystal structure of γ-iron.

4.15 List and briefly describe the four major creep deformation mechanisms active in crystalline materials.

Lattice diffusion, grain boundary diffusion, grain boundary sliding, and dislocation creep. There are sub-categories of dislocation creep as well.

4.16 Under what high temperature circumstances is a very small grain size detrimental, and under what circumstances is it advantageous?

A very small grain size is associated with rapid creep, so it would not be advantageous when creep would pose a problem for a component in service. However, it promotes ductility and is critical for superplastic forming, so it may be advantageous when fabricating a component for which creep will not ultimately be a major concern.

4.17 What do the regions on a Deformation Mechanism Map (DMM) represent?

The regions indicate a range of stress and temperature (or some other combination of parameters) over which a particular creep mechanism plays the major role in determining the creep rate. Note that other mechanisms may also be active, but they will not be the fastest for that particular set of conditions.

4.18 When designing an alloy for creep resistance, why is it generally advantageous to employ multiple composition and microstructure strategies?

Several creep mechanisms may contribute to the overall creep rate of an alloy, so slowing down a single mechanism through the use of a single composition or microstructure strategy is unlikely to be sufficient to eliminate creep as a concern.

4.19 Explain what the Larson-Miller parameter is used for, and what assumption underlies the form of the expression that makes use of this parameter.

The Larson-Miller parameter allows one to perform certain creep and/or creep rupture tests covering a convenient range of stress and temperature and then to extrapolate the data to the time-temperature-stress regime of interest that would be otherwise difficult or prohibitively expensive to test directly. Many materials are thought to exhibit a constant Larson-Miller parameter $[T(C + \log t)]$ *for a given applied stress, assuming that the activation energy* ΔH *is independent of applied stress and temperature (which is not always true).*

4.20 What are the units of temperature and time for which the typical Larson-Miller parameter is equal to ~20 for many metallic alloys?

The magnitude of the material constant C does not depend on the temperature scale but only on units of time. Since practically all data reported in the literature give both the material constant C and the rupture life in more convenient units of hours rather than in seconds—the recommended SI unit for time—test results in this book are described in units of hours.

4.21 What is the advantage in casting turbine blades that have either highly aligned grain boundaries or no grain boundaries at all?

Without grain boundaries, grain boundary sliding and diffusional creep cannot take place. This leaves only dislocation creep to deal with.

4.22 What is a TBC and what important roles does it play?

A TBC is Thermal Barrier Coating. It acts as a thermal insulator on the surface of certain components in a gas turbine, including the turbine blades. It creates a temperature difference between the combustion gas and the metal that allows a given metal to survive in an environment that would otherwise cause unacceptable creep damage.

4.23 What is a fundamental difference between the degree of recoverable strain after creep of metals and ceramics vs. many polymers?

Creep of metals and ceramics is generally most viscoplastic in nature, so it is largely unrecoverable after the load is removed. Many polymers, however, are viscoelastic over a certain temperature range, and so the creep strain may be largely recoverable. Others will have s significant viscoplastic contribution, so there is no single behavior that adequately sums up the recoverability of "most" polymers under "most" conditions.

4.24 What is the link between polymer free volume and creep rate, and why is free volume a more important concept for polymers than for metals and ceramics?

Free volume in a polymer gives the polymer chains room to slide, and thus for the material to creep. In metals and ceramics, only the grain boundary regions can be considered to have significant free volume. As such, there is usually much less freedom for rearrangement at the atomic scale. This goes a long way to explaining the greater tendency for time-dependent deformation in polymers.

4.25 What provides the driving force for rekinking of an amorphous polymer chain when loaded and unloaded?

Entropy. An amorphous high molecular weight polymer chain is highly kinked in the unloaded state. When loaded, it straightens out, lowering the entropy. When release, the entropy can be increased (which is thermodynamically favorable) by rekinking the chain.

4.26 What is an important consequence of time-temperature equivalence for testing of polymer time-dependent elastic moduli?

Because increased time and increased temperature have the same effect on polymer relaxation, and the two influences can be related to each other in a reliable fashion, it is possible to use short-time elevated temperature testing to predict the behavior of a polymer over long times at lower temperatures. This saves time and money.

4.27 What data from a standard creep plot does an isochronous diagram extract? An isometric diagram?

A standard creep plot shows strain as a function of time. Multiple curves can be plotted for different stress levels. An isochronous diagram extracts the creep strain at a certain time for multiple stress levels, and plots the data on stress-strain axes. Multiple curves can be plotted for different strain levels. An isometric diagram extracts the times needed to reach a constant level of creep strain at different stress levels, and plots the data on stress-time axes. Again, *multiple curves can be plotted for different strain levels. In all cases there are three pieces of information being depicted—strain, time, and stress—but the axes change for convenience.*

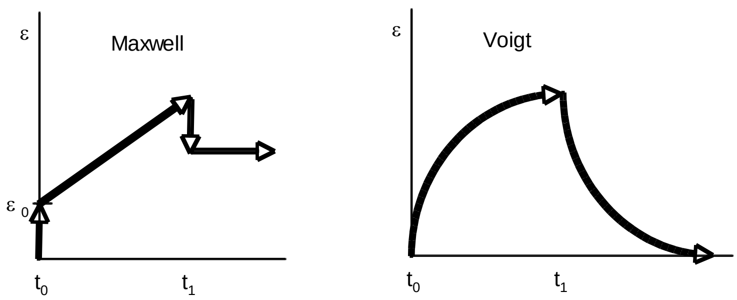
4.28 What two mechanical components are often used to model the time-dependent behavior of polymers?

A spring (a purely elastic, time-independent component) and a dashpot (a viscous element).

4.29 If a simple Maxwell model and a simple Voigt model for viscoelastic flow are loaded at time t_0, held for a certain time t_1, and then unloaded, what is different about their responses during the loading and unloading processes? And, what is different about the final state of the two mechanical models? Sketch a plot of strain vs. time for both cases to illustrate your descriptions.

A simple Maxwell model consists of a spring and dashpot in series. When a load is suddenly added, the spring element responds immediately at t_0, then the dashpot slowly extends until the hold time ends at t_1. When the load is suddenly removed, the spring element retracts immediately but the dashpot experiences no driving force so it stays extended. This combination of behaviors leaves a permanent strain in the Maxwell model.

A simple Voigt model consists of a spring and dashpot in parallel. When loaded, the spring element cannot respond immediately because it is limited by the strain rate allowed by the viscous dashpot; they must respond to the applied load together. The stiffness of the spring limits the extension of the mechanism for a fixed load. When unloaded, the same behavior occurs, with the elastic energy stored in the spring providing the driving force that gradually restores the dashpot to its original length. This combination of behaviors leaves no permanent strain in the Voigt model after unloading.



4.30 Of what phenomenon is tanδ a measure, and what does it mean when tanδ is large?

The parameter $\tan\delta$ is a measure of damping (i.e., energy loss). The δ term indicates the phase angle between the drive signal and the material response in a dynamic test like a torsional pendulum test, i.e., how far behind the applied stress signal the actual strain signal lags. The ratio of loss modulus to storage modulus, E"/E', is $\tan\delta$. Thus a large $\tan\delta$ indicates a large fraction of energy loss, or high damping.

Practice

4.31 A study of creep in ODS-Al alloys[1] found the following relationships between the minimum creep strain rate and the rupture life. Determine the Monkman-Grant constants

[1] D. C. Dunand, B. Q. Han, and A. M. Jansen, Metal. Mater. Trans. A **30**, 829 (1999).

m, B, and C_{MG} using units of hours and %/hr, then predict the rupture lifetime in hours for a minimum creep rate of 1.0×10^{-9} s^{-1}. How does this value of C_{MG} compare to typical values for other materials?

strain rate s^{-1}	t_r (ks)
3.00×10^{-8}	183.7
9.00×10^{-8}	55.3
1.00×10^{-7}	72.6
4.30×10^{-8}	133.4
5.90×10^{-9}	884
1.20×10^{-6}	13.8

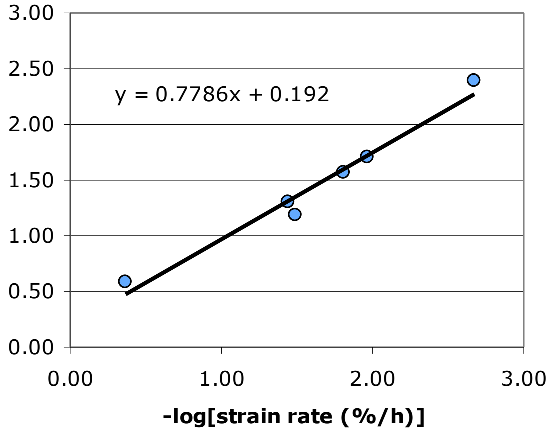
The Monkman-Grant relation is $\log t_R + m \log \dot{\varepsilon}_s = B$*, for which we have rupture time and strain rate data given. Rearranging the expression gives*

$\log t_R = -m \log \dot{\varepsilon}_s + B$*, which indicates that plotting the log of the rupture time against the log of the strain rate will give a straight line with a slope of –m and an intercept of B.*

We begin by converting the strain rates into units of %/hour and the rupture times into hours. Then we calculate the negative log of the strain rate so that the slope will be positive. This isn't necessary, just a convenience.

strain rate %/h	*t_r (h)*	*-log(strain rate)*	*log(t_r)*
0.0108	*51.03*	*1.97*	*1.71*
0.0324	*15.36*	*1.49*	*1.19*
0.036	*20.17*	*1.44*	*1.30*
0.01548	*37.06*	*1.81*	*1.57*
0.002124	*245.56*	*2.67*	*2.39*
0.432	*3.83*	*0.36*	*0.58*

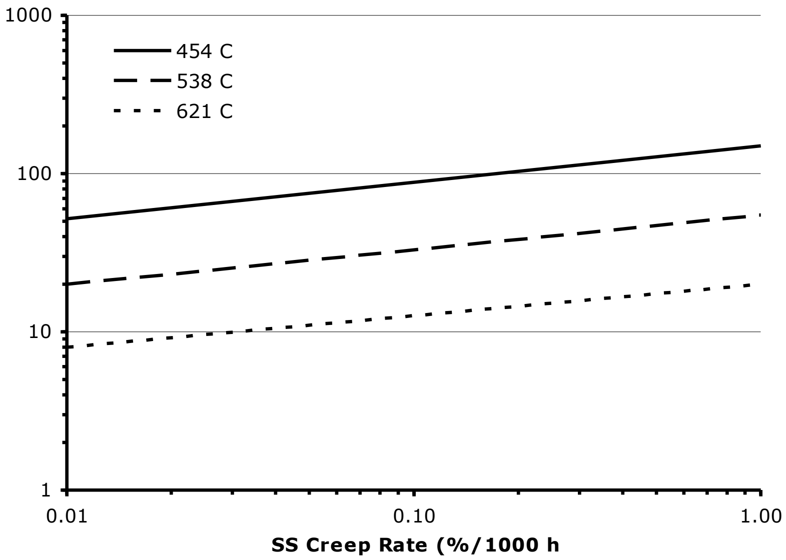
Plotting and extracting the slope and intercept gives m=0.78 and B=0.192. Converting B to C_{MG} gives C_{MG} = 1.56. This is on the low side as compared to typical values.



Finally, $t_R = \dfrac{C_{MG}}{\dot{\varepsilon}_s}$ *allows us to calculate the rupture lifetime in hours for a minimum creep rate of 1.0x10*$^{-9}$ *s*$^{-1}$ *as*

$$t_R = \frac{C_{MG}}{\dot{\varepsilon}_s} = \frac{1.56}{1\times10^{-9}s^{-1}} = 1.56\times10^{9}s = 433\times10^{3}h$$

4.32 Use the diagram below to answer the following questions.



(g) Determine the value of the creep stress exponent "n" at 454 °C (850 °F)

Start with the basic equation relating the steady state creep rate to stress and temperature:

$\dot{\varepsilon} = K\sigma^n e^{-\Delta H/RT}$ *and rewrite it for convenience as*

$$\ln(\dot{\varepsilon}) = \ln\left(K\sigma^n e^{-\Delta H/RT}\right) = \ln(K) + n\ln(\sigma) + (-\Delta H/RT)$$

$$\ln(\sigma) = \frac{1}{n}\ln(\dot{\varepsilon}) - \frac{1}{n}\left[\ln(K) + \Delta H/RT\right]$$

We must therefore read from the plot two pairs of stress and creep rate points determined at a single temperature, take the natural log of each, and calculate the ratio such that

$$\frac{\Delta\ln\sigma}{\Delta\ln\dot{\varepsilon}} = \frac{1}{n}$$

*The stresses at the far ends of the 454 °C line may be estimated from the plot as 52 MPa and 150 MPa, corresponding to strain rates of 0.01*1000=10% and 1.0*1000=1000%, so*

$$\frac{1}{n} = \frac{\Delta\ln\sigma}{\Delta\ln\dot{\varepsilon}} = \frac{\ln(150)-\ln(52)}{\ln(1000)-\ln(10)} = \frac{1.059}{4.605} = \frac{1}{4.35}$$

(h) What mechanism or category of mechanism is implied by this “n” value, diffusion or dislocation creep?

A value of n equal to 4.35 is in the range of n ≈ 4 – 5, the value associated with dislocation creep (climb of edge dislocations around barriers). It is much closer to this value than n=1 for diffusion creep, either Nabarro-Herring or Coble.

(i) Without calculating “n” for the other two temperatures, is “n” a strong function of temperature? How do you know?

Not a strong function of temperature because the slopes are all similar.

(j) Determine the activation energy for creep in units of kJ/mol, assuming that “n” is not a strong function of temperature (regardless of reality).

Imagine plotting the natural log of the strain rate vs. 1/T for constant stress. The slope of this curve will be -ΔH/R. Although several different choices work, a stress of 20 MPa is convenience because it corresponds to 538 °C at a SS creep rate of 0.01 %/1000 h, and to 1.00 %/1000 h at 621 °C. Thus

$$-\frac{\Delta H}{R}=\frac{\Delta \ln \dot{\varepsilon}}{\Delta\left(\frac{1}{T}\right)}=\frac{\ln(1000)-\ln(10)}{\frac{1}{621+273}-\frac{1}{538+273}}=\frac{4.605}{-1.145\times 10^{-4}}=-40226\ K$$

$$\Delta H=-R\left(-40226\ K\right)=\left(8.3144\ J/mol\cdot K\right)\left(40226\ K\right)=334457=334.4\ kJ/mol$$

4.33 For a certain high-temperature alloy, failure was reported after 4100 hrs at 680°C when subjected to a stress level of 270 MPa. If the same stress were applied at 725°C, how long would the sample be expected to last? State any assumptions you must make to allow you to make this determination.

This is a case of using information about the rupture life at one temperature and stress level to predict the rupture life at a different temperature but the same stress level. When this is the case, the Larson-Miller parameter is quite useful. However, we must assume that the same mechanisms are responsible for the creep failure at both temperatures, and we must assume a value for the Larson-Miller parameter. A value of 20 is a good approximation for many metallic materials. This allows us to say:

ΔH/R = T(C + logt) = T(20 + logt) = constant

We can equate the Larson-Miller relation for the two cases, then solve for the rupture life at 725 °C.

(680 °C + 273)(20 + log(4100)) = (725 °C + 273)(20 + log(t))

$$\log t=\frac{953\left(20+\log(4100)\right)}{998}-20$$

$$\log t=2.548$$

$$t=353\,h$$

Clearly the higher temperature has had a profound influence on the rupture life!

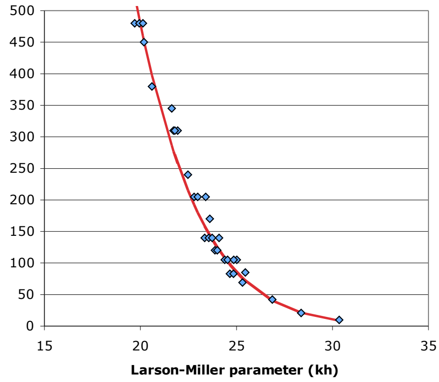
4.34 Construct a Larson-Miller plot using the following creep rupture data, assuming C=20.

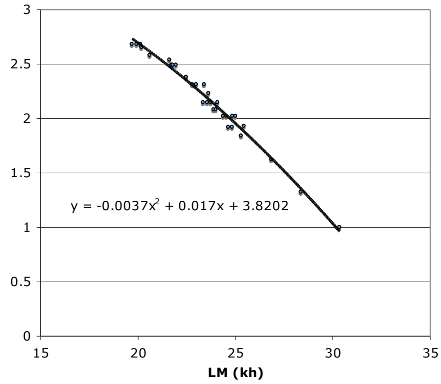
Temp. (°C)	Stress (MPa)	Rupture Time (hr)	Temp. (°C)	Stress (MPa)	Rupture Time (hr)
650	480	22	815	140	29
650	480	40	815	140	45
650	480	65	815	140	65
650	450	75	815	120	90
650	380	210	815	120	115
650	345	2700	815	105	260
650	310	3500	815	105	360
705	310	275	815	105	1000
705	310	190	815	105	700
705	240	960	815	85	2500
705	205	2050	870	83	37
760	205	180	870	83	55
760	205	450	870	69	140
760	170	730	870	42	3200
760	140	2150	980	21	440
			1095	10	155

(k) Plot the data twice: (i) with axes of Stress vs. the LM parameter in thousands of hours, and (ii) with axes of Log(Stress) vs. the LM parameter in thousands of hours. Fit a second-order polynomial to the second plot.

A Larson-Miller plot is Stress vs. the Larson-Miller parameter, T(C + logt). For convenience, often this is plotted as log(stress) and as T(C + logt) in units of thousands of hours. In this case, we've actually calculated the log(stress) axis rather than plotting the linear data on a log axis. This made the polynomial fit straightforward to determine directly. The inverse log of the polynomial was then used to add the fitted curve to the linear plot on the left for use in later parts of this problem.

$$\sigma = 10^{-0.0037(LM)^2 + 0.017(LM) + 3.8202}$$





Once the plot with the fitted curve exists, the L-M parameter can be determined for a chosen stress level, then it can be set equal to T(20 + logt) for the temperature of interest.

(l) Determine the expected life for a sample tested at 650°C with a stress of 240 MPa, and at 870°C with a stress of 35 MPa.

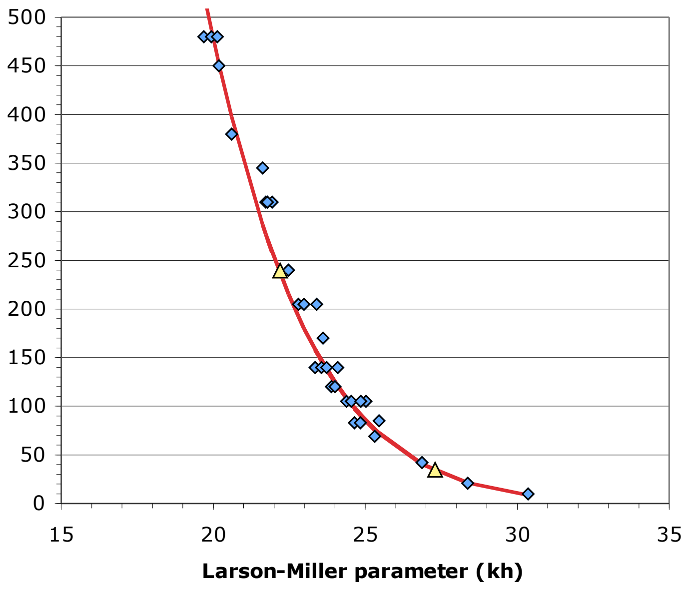
Case 1: The LM parameter at 240 MPa can either be determined graphically from the fitted curve, or numerically using the quadratic equation. Here we've opted to do it graphically. Then we plug in 650 °C for the temperature, converting to Kelvin units.

$$LM = T\left(C + \log t\right)$$
$$\log t = \frac{LM}{T} - C = \frac{22200}{650°C + 273} - 20$$
$$\log t = 4.052$$
$$t = 11272\ h$$

Case 2: The LM parameter at 35 MPa can either be determined graphically from the fitted curve, or numerically using the quadratic equation. Again we've opted to do it graphically.

$$LM = T\left(C + \log t\right)$$
$$\log t = \frac{LM}{T} - C = \frac{27300}{870°C + 273} - 20$$
$$\log t = 3.885$$
$$t = 7665\ h$$

The two points have been added to the linear plot to show that they fall on the curve.



(m) What is the maximum operational temperature such that failure should not occur in 5000 hr at stress levels of 140 and 420 MPa, respectively?

Extract the LM parameter for the stress levels of interest. Then use the same expression as above but rearrange to solve for temperature. For stress equal to 140 MPa

$$LM = T\left(C + \log t\right)$$

$$T = \frac{LM}{C + \log t} = \frac{23700}{20 + \log\left(5000\right)} = 1000\ K$$

$$T = 727\ °C$$

For stress equal to 420 MPa

$$LM = T\left(C + \log t\right)$$

$$T = \frac{LM}{C + \log t} = \frac{20400}{20 + \log\left(5000\right)} = 861\ K$$

$$T = 588\ °C$$

4.35 A 200-mm-long polypropylene rod, with a rectangular cross-section that is 20 mm by 4 mm, is subjected to a tensile load of 300 N, directed along its length. If the rod extends by 0.5 mm after being under load for 100 s, determine the creep modulus. How does this value compare with the typical static modulus for polypropylene, and what does the comparison imply about the amount of creep that has probably taken place in 100 s?

The creep stress is given by

$$\sigma_0 = \frac{P}{A} = \frac{300\ N}{20 \times 10^{-3}\left(4 \times 10^{-3}\right)} = 3.75\ MPa$$

The creep strain after 100 seconds under load is

$$\varepsilon_{100} = \frac{0.5 \times 10^{-3}}{200 \times 10^{-3}} = 0.0025$$

Then the creep modulus is given by

$$E_C = \frac{\sigma_0}{\varepsilon\left(t\right)} = \frac{3.75 \times 10^6}{0.0025} = 1.5\ GPa$$

This compares favorably with the static modulus of 1.1-1.6 GPa given in Chapter 1. This implies that little creep strain has developed in the 100 s of loading, and that most of the strain is elastic.

4.36 Calculate the typical relaxation time for silicate glass and comment on its propensity for stress relaxation at room temperature. E ≈ 70 GPa and η ≈ 1 × 1012 GPa-s (1022 poise).

$$Ŧ = \frac{\Omega}{E} = \frac{10^{12}\ GPa - \sec}{70\ GPa} = 1.40 \times 10^{10}\ sec$$

There is very limited stress relaxation. The material will behave in essentially elastic fashion.

4.37 The deformation response of a certain polymer can be described by the Voigt model. If E = 400 MPa and $\eta = 2 \times 10^{12}$ MPa-s, compute the relaxation time. Compute $\varepsilon(t)$ for times to $5Ŧ$ when the steady stress is 10 MPa. How much creep strain takes place when $t = Ŧ$ and when $t = \infty$?

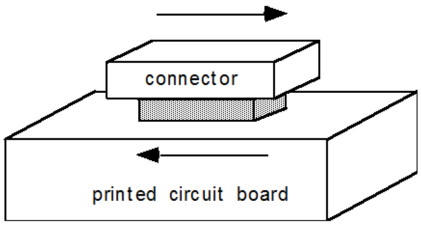
$$Ŧ = \frac{\Omega}{E} = \frac{2 \times 10^{12} MPa - sec}{400\ MPa} = 5 \times 10^{9}\ sec$$

$$\epsilon = \frac{\sigma_0}{E}\left(1 - e^{-\frac{t}{Ŧ}}\right)$$

Time	ϵ
$t = Ŧ$	1.58×10^{-2}
$t = 5Ŧ$	2.48×10^{-2}
$t \to \infty$	2.50×10^{-2}

Design

4.38 A solder joint between a computer chip connector and a printed circuit board is typically subjected to shear due to thermal expansion differences. For optical communications devices (e.g., those that use lasers to transmit information), dimensional stability is critical or the components will lose alignment. A crude approximation of a circuit board, solder joint, and connector is shown here with the shear load indicated by arrows. For this problem, assume that the critical joint is 0.5 mm thick, 2 mm wide and 4 mm long.



(n) If the component is designed to last for five years under ordinary use, which is likely to be more important solder data: steady state creep rate or rupture life? Explain briefly.

Steady state creep rate because (i) it allows calculation of strains over long periods of time, and (ii) failure in this case is defined as a certain amount of strain and not by fracture or loss of structural integrity.

(o) It is found that accelerated creep tests of a particular solder give the following results. If it is known that the stress exponent for this particular solder is n = 6, calculate the steady state creep rate at a stress level of 100 MPa and a temperature of 100°C.

Shear Stress (MPa)	Strain rate (s^{-1})	T (°C)
70	1 x 10^{-5}	190
70	2.5 x 10^{-3}	225

Start with the basic equation, but interpret the stress and strain as shear stress and shear strain. The form will remain unchanged:

$$\dot{\varepsilon} = K\sigma^n e^{-\Delta H/RT}$$

Then rewrite it for convenience as

$$\ln(\dot{\varepsilon}) = \ln\left(K\sigma^n e^{-\Delta H/RT}\right) = \ln(K) + n\ln(\sigma) + (-\Delta H/RT)$$

Both temperature and stress level are changing, so K and ΔH must be determined.

$$-\frac{\Delta H}{R} = \frac{\Delta \ln \dot{\varepsilon}}{\Delta\left(\frac{1}{T}\right)} = \frac{\ln(1\times10^{-5}) - \ln(2.5\times10^{-3})}{\frac{1}{190+273} - \frac{1}{225+273}} = \frac{-5.52}{1.518\times10^{-4}} = -36365\ K$$

$$\Delta H = -R(-36365\ K) = (8.3144\ J/mol\cdot K)(36365\ K) = 302352 = 302.4\ kJ/mol$$

$$K = \frac{\dot{\varepsilon}}{\sigma^n e^{-\Delta H/RT}} = \frac{1\times10^{-5}}{70^6 e^{-36365/(190°C+273)}} = 1.096\times10^{18}$$

Finally

$$\dot{\varepsilon} = K\sigma^n e^{-\Delta H/RT} = (1.096\times10^{18})(100^6)e^{-36365/(100°C+273)}$$
$$\dot{\varepsilon} = 3.6\times10^{-13}\ s^{-1}$$

(p) What *assumption* did you make about the nature of the creep at 100, 190 and 225°C in order for the calculation in part "b" to be possible? How could you check the validity of this assumption experimentally?

Had to assume that the creep mechanism is unchanged over the range of stress and temperature selected. This could be verified by checking the slope of a natural log strain rate vs. 1/T plot to see if the slope is constant.

(q) If the maximum allowable shear displacement of the connector relative to the printed circuit board after 3 years of continuous use is 0.25 μm, will the joint meet the design criteria?

3 years = $9.46\text{x}10^7$ s

$$\Delta l = l_0\varepsilon = l_0\dot{\varepsilon}t = (0.5\ mm)(1\times10^{-13}\ s^{-1})(9.46\times10^{7}\ s) = 4.73\times10^{-6}\ mm = 4.76\times10^{-3}\ \mu m$$

This is well below 0.25 μm so the design meets the creep criterion.

(r) Possible solder materials for this application include Indium, Lead-Tin alloy, Bismuth-Tin alloy, and Tin-Silver-Copper alloy. Without doing any mechanical testing or looking up any mechanical data on these metals, how could you make a reasonable attempt at rank ordering them from slowest creep rate to fastest creep rate at 100°C? Don't actually make the list, just explain what information you would need and how you would use it.

Check the melting point of each material to determine the homologous temperature for each at the operating temperature. To a first approximation, we might guess that the solder with the lowest homologous temperature would creep the least. As we know, there is more to creep than homologous temperature, however, so this would only use useful as a first guess.

4.39 A 10-cm-long rod of PVC is used to connect two stiff plates that are a fixed distance apart. The fastening process causes the rod to extend by 0.15 cm. Given that the elastic modulus of PVC is 3 GPa and assuming that the material's creep behavior is similar to that characterized by the data shown in Figure 4.44, estimate the initial stress on the rod and the stress after 30 days.

Since the extended length of the rod is fixed (i.e., 10.15 cm), this problem really requires stress relaxation data, not creep data (for which stress, not strain, is fixed). However, we may reasonably estimate both the initial stress and that after 30 days from the creep data by using the isometric stress-time curve shown in Fig. 6.19c. To begin, we note that the rod experiences a strain of

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{0.15}{10} = 0.015$$

From the stress-time plot at a strain of 0.015, we estimate the initial stress to be 41-42 MPa, corresponding to a one second load application. By comparison, if we assume that the initial stress is purely elastic, then from Hooke's law:

$$\sigma = E\varepsilon = 3\times10^{9}(0.015) = 45\ MPa$$

By comparison, the stress values determined by the two methods are in relatively good agreement, though some evidence for viscoelastic deformation is already present after only one second of load application.

Proceeding further, we find from the stress-time curve, corresponding to a strain of 0.015 that the stress after 30 days (2.6x10^{6} seconds) has dropped by approximately 45% to 23 MPa.

4.40 A superalloy gas turbine component was originally designed to operate at temperatures up to 760 °C and exhibited a stress rupture life of 900 h under this operating condition. An updated design calls for a thermal barrier coating to be added to the same component to allow engine operation under the same conditions, but with an increase in reliability. If an increase in rupture life to 1800 h is desired, what temperature differential between the inside and outside of the component must be achieved by the addition of the TBC?

Use the Larson-Miller relation with an assumption that C=20.

$$LM = T_1\left(C + \log t_1\right) = T_2\left(C + \log t_2\right)$$

$$T_2 = \frac{T_1\left(C + \log t_1\right)}{\left(C + \log t_2\right)} = \frac{\left(760\ °C + 273\right)\left(20 + \log(900\ h)\right)}{\left(20 + \log(1800\ h)\right)} = \frac{23712}{23.26} = 1019\ K = 746\ °C$$

$$\Delta T = T_2 - T_1 = 746\ °C - 760\ °C = -14\ °C$$

4.41 A 10-cm-long cylindrical rod of polypropylene is subjected to a tensile load of 550 N. If the maximum allowable strain of 2% is experienced no earlier than after four months of service, what is the minimum required rod diameter? Also, what is the rod diameter after the four-month service period? You will find the plot in the next problem to be useful.

Four months of service corresponds to approximately 10^7 seconds. From the graph associated with the next problem, we see that the maximum allowable stress to limit strain to 2% after 10^7 seconds is 7 MPa. It follows that

$$A = \frac{P}{\sigma}$$

$$\pi\left(\frac{d}{2}\right)^2 = \frac{550\ N}{7 \times 10^6}$$

$$d = 10\ mm$$

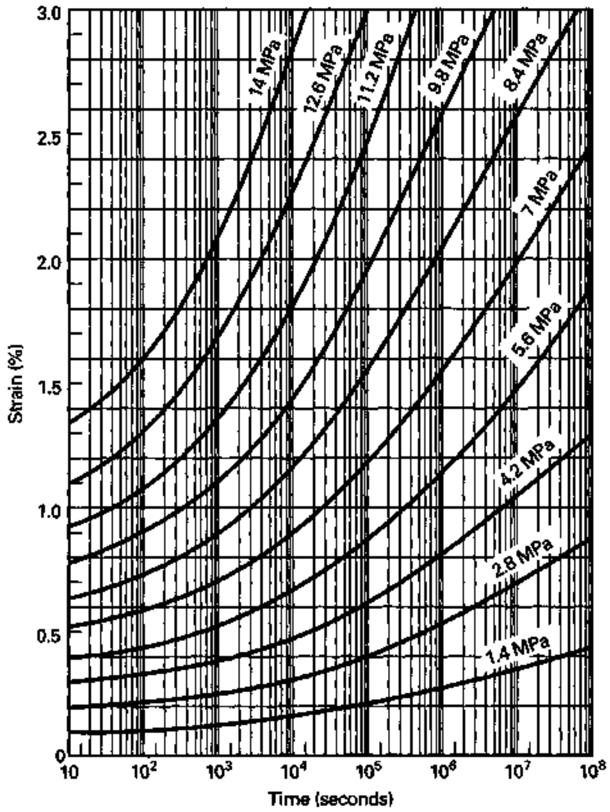
Assuming that plastic deformation (including viscoplastic deformation from creep) involves a constant volume process, we see that

$$A_1 \ell_1 = A_2 \ell_2$$

$$d_1^2 \ell_1 = d_2^2 \ell_2$$

$$d_2^2 = \frac{d_1^2 \ell_1}{\ell_2} = \frac{\left(10 \times 10^{-3}\right)^2 \left(10 \times 10^{-2}\right)}{10.2 \times 10^{-2}} = 9.9\ mm$$

4.42 For safe and reliable operation, a certain polypropylene pipe must withstand an internal pressure of 0.5 MPa for a minimum of three years. If the pipe diameter is 100 mm, what is the minimum necessary wall thickness to ensure that the pipe will not experience a strain greater than 1.3%? To solve this problem, use the accompanying graph that reveals the room temperature creep response for polypropylene. (R. J. Crawford, Plastics Engineering, SPE, Brookfield Center, CT (1981). Reprinted by permission.)



From the graph, we see that the allowable stress for a three-year service period (nearly 10^8 s) associated with a maximum strain of 1.3% is 4.2 MPa. The minimum required pipe wall thickness is then calculated to be

$$\sigma = \frac{PR}{t} = \frac{PD}{2t}$$

$$t = \frac{PD}{2\sigma} = \frac{(0.5\ MPa)(100\ mm)}{2(4.2\ MPa)} = 5.95\ mm$$

Extend

4.43 What are the dictionary definitions and the etymologies of the words *isochronous* and *isometric*?

Isochronous is "occurring at the same time", from the Greek isos "equal" and chronos "time". Isometric is "of, pertaining to, or having equality of measure", from the Greek isos "equal" + metron "measure".

4.44 Using information acquired from the National Transportation Safety Board (NTSB), explain the role that epoxy creep played in the 2006 collapse of a 3-ton concrete ceiling panel in Boston's Fort Point Channel Tunnel. What could have been done to prevent this disaster?

The NTSB found that the epoxy used to affix the support bolts to the tunnel ceiling was not chosen for creep resistance. Creep in the epoxy adhesive allowed many of the bolts to pull out of the ceiling, leaving insufficient support for the heavy concrete panel. Recognition that creep would be important for this application and specifying a creep-resistant epoxy would likely have preventing this disaster.

CHAPTER 5

Review

5.1 Why do most materials exhibit fracture strengths much lower than their theoretical capacities to support load?

Real materials have flaws that cause stress concentrations at which the local stress is much greater than the global stress. The applied stress is therefore much lower than the theoretical strength at the point of failure.

5.2 If a set of small ceramic bars is tested to failure with the goal of predicting the service failure probability of a larger ceramic component, how does the probability of failure at a given stress level compare for the two components?

Because there is a greater statistical probability of a large flaw in a specimen with a larger surface area, the probability of failure at a given stress level is greater for the larger ceramic component than for the small test specimens.

5.3 Is it preferable to have a larger or smaller Weibull modulus *m* for a structural product?

A small Weibull modulus indicates large distributions in failure, which makes for unreliable products. Conversely, a large Weibull modulus indicates consistent components with a small failure distribution. The latter case is greatly preferable, so a large Weibull modulus is preferred.

5.4 Name, define, and give an expression for k_t.

The term $2\sqrt{a/\rho}$ *is defined as the stress-concentration factor* k_t *and describes the effect of crack geometry on the local crack-tip stress level. It describes the amplification of stress on a local scale due to the presence of a flaw.*

5.5 Write the equation that relates *applied stress* to *local maximum stress*.

$\sigma_{max} = \sigma_a(1+2\sqrt{a/\rho})$ and if $a \gg \rho$ then $\sigma_{max} \approx 2\sigma_a\sqrt{a/\rho}$

5.6 Eq. 5-26 can often be simplified by assuming that $a \gg \rho$. For what ratio of a/ρ can one make this assumption if the error introduced in σ_{max} must be less than 1%? If the crack tip radius is on the order of 10 nm, what is the corresponding minimum crack length?

In order for the error to be less than 1%, the values of the exact solution and the approximate solution must be within 99% of each other. From the following analysis, it can be seen that the crack length must be greater than the crack tip radius by a factor of 2500 or more to ensure an error of less than 1%. If the crack tip radius is approximately 10 nm then the minimum crack length would be 25000 nm, or 25 µm.

$$\frac{2\sqrt{a/\rho}}{1+2\sqrt{a/\rho}} = 0.99$$

$$(1.01)(2\sqrt{a/\rho}) = 1+2\sqrt{a/\rho}$$

$$2\sqrt{a/\rho} = \frac{1}{0.01} = 100$$

$$a/\rho = 50^2 = 2500$$

5.7 Explain why boring a hole in a part may extend its lifetime.

Boring a hole at the tip of a crack can artificially increase the radius of the crack tip, thereby reducing the stress concentration. This, in turn, decreases the likelihood of the local stress exceeding the critical value for fast fracture (or creating conditions that favor rapid fatigue crack growth).

5.8 Give three examples of design features that may lead to a reduction in component fracture strength.

Examples include: small holes drilled in directions perpendicular to the primary tensile load, sharp corners at the base of a gear tooth, sharp concave threads on a bolt, rapid transitions in size along the length of a cylinder such as between a bolt shaft and a bolt head, identification marks stamped on the surface of a component, etc.

5.9 Explain the connection between notch strengthening and material ductility; which is more likely to notch strengthen, pure Al or martensitic steel?

If a large amount of plastic deformation is possible then the high stress at the tip of a crack can cause plasticity, rapid cross-section shape change, and the development of a triaxial stress state. This leads to greater resistance to further plastic deformation than might be expected based on the net section stress. Pure Al has low yield strength and lots of capacity to harden, so it is likely to notch strengthen. Martensitic steel has very high strength and little capacity for plasticity, so it is not a good candidate for notch strengthening (and will be much more likely to notch weaken).

5.10 Identify the conditions under which a brazed joint is likely to be "stronger" than the brazing material by itself. Describe the trends in joint fracture strength as it becomes wider/narrower or thinner/thicker.

The onset of plastic deformation in a brazed joint can be delayed by the constraint of the materials being joined. If the joint is very thin or wide so that the aspect ratio of width:thickness is very large, the constraint will be maximized and the strength will also be maximized.

5.11 List the *microscopic* fracture surface markings for metals, state under what typical conditions each is produced, and identify a visible characteristic associated with each mechanism that would allow you to identify it from a fracture surface micrograph.

Slant fracture (or shear lips) are fracture surfaces oriented approximately 45° to the tensile axis. They appear when there is plastic deformation involved in the fracture process.

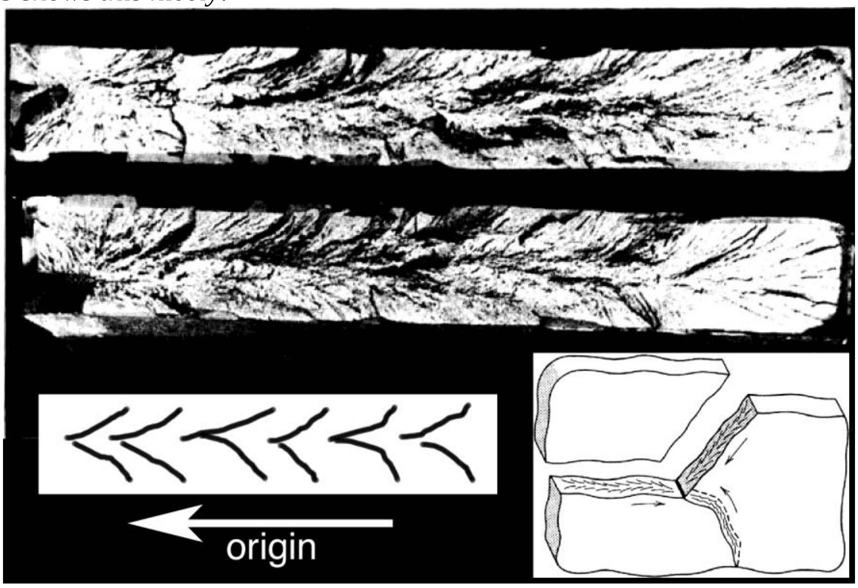
Tear marks are smears on otherwise flat surfaces that indicate highly ductile fracture in shear or torsion.

Radial marks are lines fanning out on a fracture surface, emanating from the fracture origin site. They are macroscopic evidence of relatively brittle (fast) metal fracture.

Chevron marks are like radial marks, but confined by a thin plate. They consist of arrow-like markings that point back toward the crack origin, much like radial marks do.

5.12 Sketch a typical fracture surface of a steel plate that displays chevron markings; identify the origin of the crack on the sketch.

Figure 5.18 shows this nicely:



5.13 In what class of material is stress whitening often seen, and what is the physical origin of the phenomenon?

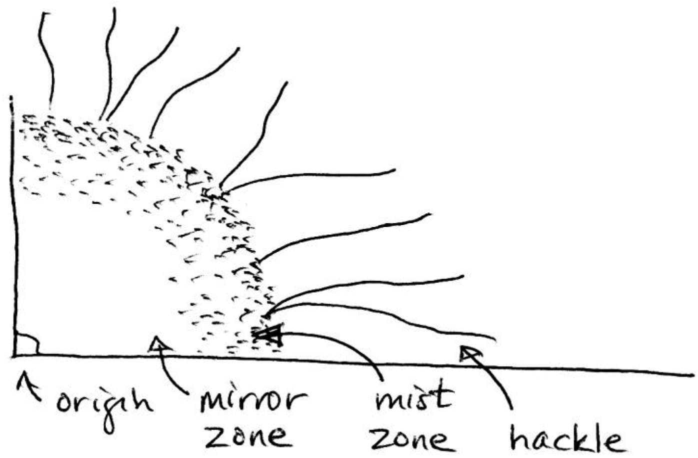
Stress whitening — induced scattering of light associated with microscopic damage — may be visible macroscopically after plastic deformation or fracture. Crazing and shear banding micromechanisms can both cause this visual effect.

5.14 What does the presence of significant crack branching in a glass fracture indicate about the fracture event?

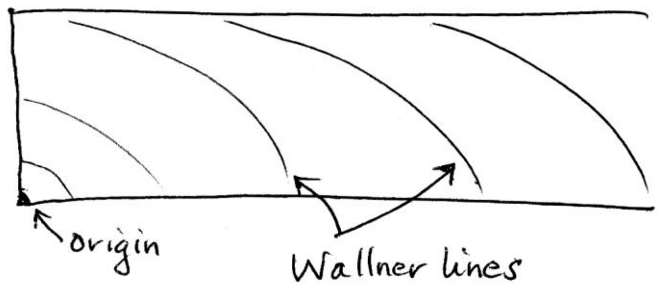
Significant crack branching in a glass fracture indicates a high-energy failure involving a strong material that was subjected to a high level of stress.

5.15 Define the following terms, provide a sketch of each, and state for what material or materials they are relevant: mirror, mist, hackle, Wallner lines.

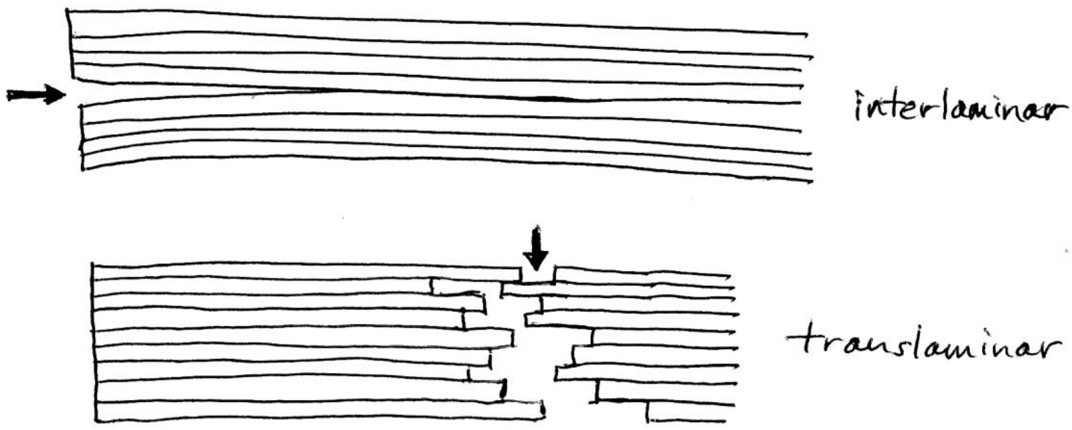
Mirror, mist, and hackle zones are regions often found on the fracture surfaces of glass, ceramic, or glassy polymer specimens. They appear in that order starting at the fracture origin point. Depending on the size of the specimen and the specimen strength, all three zones may be visible or only the first one or two may be apparent.



Wallner lines may appear inside the mirror zone, particularly when the component has two surfaces in close proximity, such as in a glass plate. They indicate the general direction of crack advance, and can be used to trace the fracture back to the point of origin.



5.16 Sketch interlaminar and translaminar fractures of a laminated composite, clearly showing the relative orientation of the crack growth and the layers in both cases.



5.17 Is the presence of micro-void coalescence on a metal fracture surface inconsistent with macroscopic evidence of brittle fracture?

Not necessarily. MVC indicates microscopic plasticity processes that are often associated with high-toughness fractures at the macroscale, but plasticity that is limited on the macroscale due to geometric constraints can still occur at the microscale if the material is inherently ductile. In the latter case, the macroscopic fracture could be brittle in appearance (e.g., a high ratio of flat fracture to slant fracture) but still exhibit plasticity on the microscale.

5.18 Identify a likely cause of intergranular fracture in metals or ceramics, and sketch a cross section of an intergranular fracture surface.

Regardless of the material class, intergranular fracture is due to weaknesses along the grain boundaries of crystalline materials (e.g., impurities or glassy boundary phases).

5.19 What role can craze formation play in toughening of an amorphous polymer? What role can crazing play in weakening an amorphous polymer?

The crazing process dissipates energy as it occurs, which makes it a toughening mechanism. Furthermore, it keeps the material intact as long as the fibrils spanning the craze are unbroken. However, the weakened path created by a craze can serve as a relatively easy crack growth path, so a polymer with preexisting crazes is likely to be weaker than one that has not yet experienced any craze formation.

5.20 Which microscopic fracture mechanism is associated with *river patterns*, and why do they form?

River patterns are associated with transgranular fracture of crystalline metals and ceramics. They form when a crack advances from one grain to another, and seeks a low-energy path along a preferred cleavage plane. The adjustment of the crack from one grain to the next creates the characteristic river patterns.

5.21 When a fiber-reinforced polymer matrix composite plate is fractured in bending, what evidence may exist that would indicate which side of the plate was in tension and which side was in compression?

Fiber pull-out requires tension, so it would only appear on the tensile side of the plate. On the compressive side of the plate there will be little or no pull-out. Instead, there may be evidence of fiber buckling.

5.22 What visual evidence supports the presence of significant grain boundary sliding in many creep failures?

So-called fiduciary marks (scratches or other features that span multiple grains) displace on either side of a grain boundary that has undergone sliding. The displacement of the two halves of a previously-continuous mark clearly indicates that the adjacent grains have slid in opposite directions.

Practice

5.23 Using data from Chapter 1, calculate the theoretical strengths of diamond and silicon carbide. If the experimentally-determined tensile strengths of certain diamond and silicon carbide fibers are 1000 MPa and 500 MPa, respectively, what is the ratio of Young's modulus to fracture strength for each material? How do they compare with the materials listed in Table 5.1? On the basis of this comparison, what conclusion might you come to regarding the quality of the diamond and silicon carbide fibers in question?

Eq. 5-7 shows that the theoretical strength may be estimated as $\sigma_c \approx E/^{1}$ *. So for diamond and silicon carbide, data from Table 1.1a we find:*

$$\sigma_{c,diamond} \approx \frac{965\,GPa}{\pi} \approx 307\,GPa = 307\times10^3\ MPa$$

$$\sigma_{c,SiC} \approx \frac{470\,GPa}{\pi} \approx 150\,GPa = 150\times10^3\ MPa$$

Both values greatly exceed the experimental measurements of 1.0 GPa and 0.5 GPa. The actual ratios of Young's modulus to fracture strength for these materials are

$$\left(\frac{E}{\sigma_f}\right)_{diamond} = \frac{965\,GPa}{1\,GPa} = 965$$

$$\left(\frac{E}{\sigma_f}\right)_{diamond} = \frac{470\,GPa}{0.5\,GPa} = 940$$

Both ratios are much higher than those shown in Table 5.1. One might conclude that the quality of the particular diamond and silicon carbide fibers tested was not very high.

5.24 When a failure data set for a ceramic material processed in a certain facility is analyzed, it is found that the characteristic strength is 327 MPa and the Weibull modulus is 8.75. A nominally identical batch of material processed in a different facility is also tested and found to have essentially the same characteristic strength, but the Weibull modulus is 6.25. At what stress level is the probability of failure equal to 50% for each set of material? What initial conclusion might you draw about the quality control procedures at the two facilities?

$$\sigma_{F1} = (F)^{1/m}\sigma_o = (0.5)^{1/8.75}\left(327\ MPa\right) = 302\ MPa$$
$$\sigma_{F2} = (F)^{1/m}\sigma_o = (0.5)^{1/6.25}\left(327\ MPa\right) = 270\ MPa$$

The material from facility 2 is likely to experience 50% failure at a lower stress than the material from facility 1. We might conclude that the quality control procedures in facility 2 are not as good as those in facility 1.

5.25 A thin plate of a ceramic material with E = 225 GPa is loaded in tension, developing a stress of 450 MPa. Is the specimen likely to fail if the most severe flaw present is an internal crack oriented perpendicular to the load axis that has a total length 0.25 mm and a crack tip radius of curvature equal to 1 μm?

By comparing the local stress at the crack tip to the theoretical strength of the material we can determine whether or not fracture is likely. The theoretical strength is given by

$$\sigma_{c,ceramic} \approx \frac{225\,GPa}{\pi} \approx 71.6\,GPa = 71.6\times 10^3\ MPa$$

The local stress is given by

$$\sigma_{max} = \sigma_a(1+2\sqrt{a/\rho}) = \left(450\ MPa\right)\left(1+2\sqrt{250\mu m/1\mu m}\right) = \left(450\ MPa\right)\left(32.6\right) = 14.67\ GPa$$

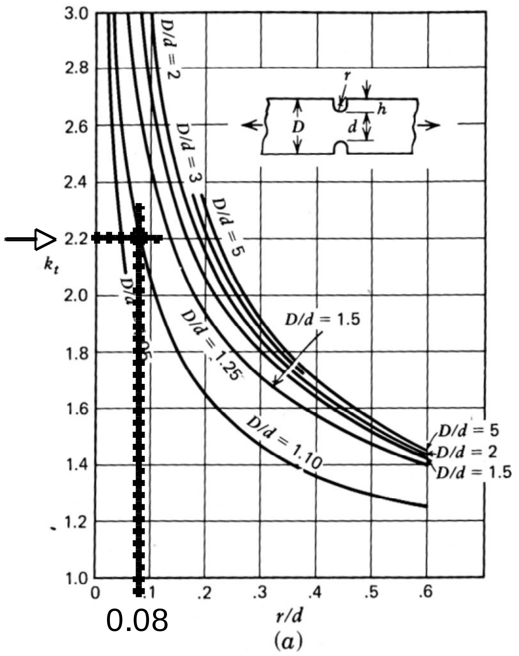
In this case, the maximum local stress (14.67 GPa) is substantially lower than the theoretical strength (71.6 GPa) so it is very unlikely that fracture will occur.

5.26 A rectangular bar is notched on two sides as shown in Fig. 5.9a. The dimensions of the bar are thickness *t*=0.2 cm, *D*=2 cm, *d*=1.8 cm, *h*=0.1 cm, and *r*=0.15 cm. If an elastic load of 15 kN is applied along the axis of the bar, what is the maximum stress in the vicinity of the notches? If the yield strength of the material is 950 MPa, will the material yield near the notches under this load?

Begin by calculating the nominal stress on the bar:

$$\sigma_a = \frac{P}{A_0} = \frac{15\times 10^3\,N}{\left(0.2\times 10^{-2}\,m\right)\left(2\times 10^{-2}\,m\right)} = 375\,MPa$$

Then determine k_t from Fig. 5.9a for the ratio of r/d=0.15/1.8=0.083 and the ratio of D/d=2/1.8=1.11. The closest line is D/d=1.10. A reasonable estimate for k_t is 2.2, as shown here



(*a*)

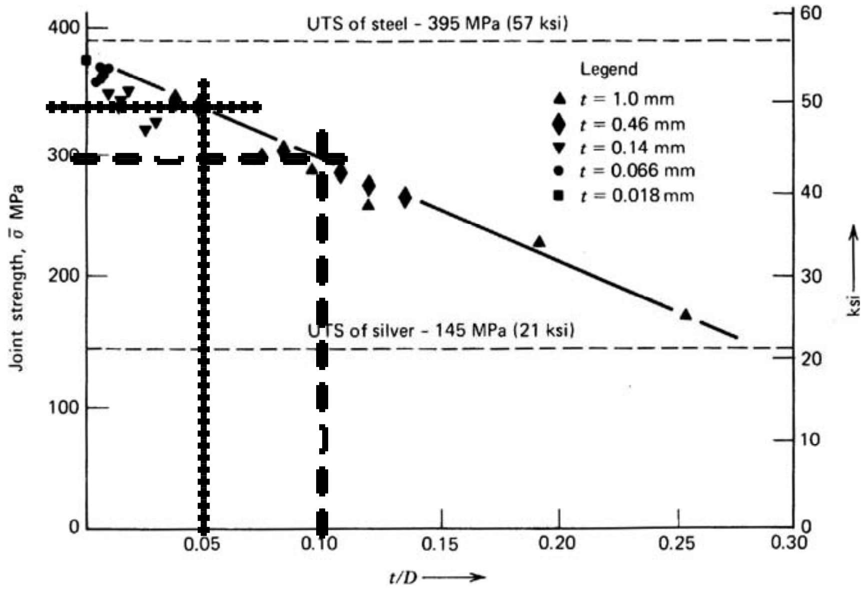
Finally, calculate the maximum local stress using k_t as follows.

$$\sigma_{\max} = \sigma_a(1+2\sqrt{a/\rho}) = \sigma_a(1+k_t) = (375\ MPa)(1+2.2) = 1200\ MPa$$

This value greatly exceeds the yield strength, so the material will yield locally under this load.

5.27 Two 0.5 cm-diameter rods of 1020 steel (σ_{ts} = 395 MPa) are to be joined with a silver braze alloy 0.025 cm thick (σ_{ys} = 145 MPa) to produce one long rod. The ultimate strength of this brazed structure is found to be approximately 345 MPa. If it is necessary to reduce the diameter of the rods to 0.25 cm with no change to the braze joint itself, will the strength (in MPa units) increase, decrease, or remain the same? If it changes, by how much?

Fig. 5.11 shows the strength of an identical steel/silver/steel braze joint as a function of joint thickness t and rod diameter D. For the initial case in which the ultimate strength of this brazed structure is found to be approximately 345 MPa, the ratio of t/D=0.025/0.5=0.05. A reduction in rod diameter D would increase the ratio t/D, so the strength of the silver braze layer would be expected to decrease compared to the original case.



At the new ratio of t/D=0.025/0.25=0.10, the expected joint strength is approximately 295 MPa. This is a strength reduction of about 50 MPa.

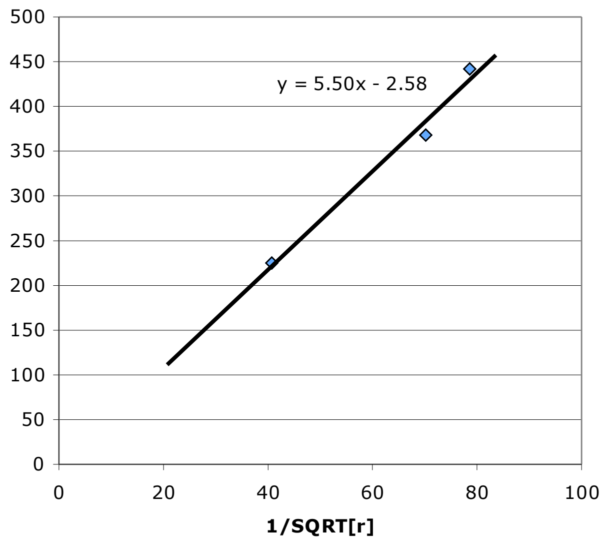
5.28 You are called as an expert witness to analyze the fracture of a sintered silicon carbide plate that was fractured in bending when a blunt load was applied to the plate center. Measurement of the distance between the fracture origin and the mirror/mist boundary on the fracture surface gives a radius of 0.796 mm. You are given three pieces of the same SiC to test, and you determine that the mirror radius is 0.603, 0.203, and 0.162 mm for bending failure stress levels of 225, 368, and 442 MPa, respectively. What is your estimate of the stress present at the time of fracture for the original plate?

The first task is to determine a mirror constant for this particular SiC material. The relationship between applied stress, radius of the mirror-mist boundary, and the mirror constant, is

$r_m = \left(M_m/\sigma\right)^2$ *that can be rearranged to give*

$$\sigma = \frac{M_m}{\sqrt{r_m}}$$

A plot of failure stress (y axis) vs. the inverse square root of the mirror radius should therefore give a straight line with a slope of M_m, the mirror constant. Sure enough, this is the case for the SiC tests and the mirror constant is approximately 5.5 MPa $\sqrt{m}$.



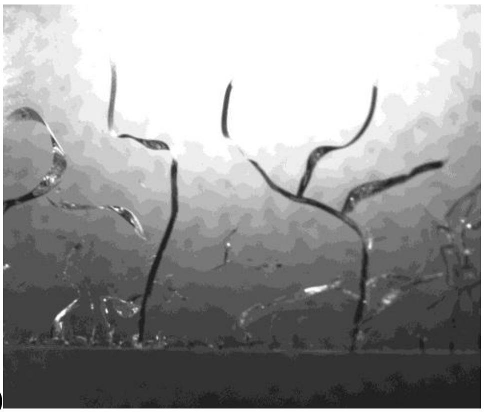
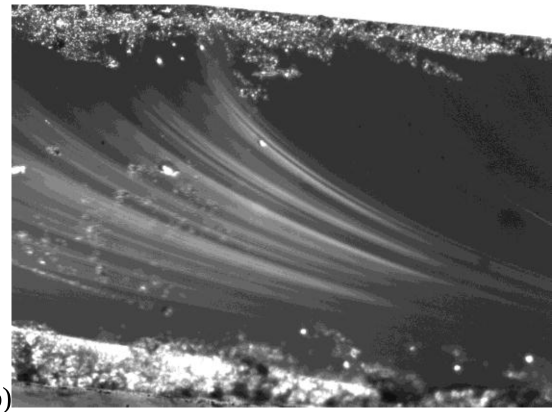
With this information in hand, the failure stress for the SiC plate is estimated to be

$$\sigma = \frac{M_m}{\sqrt{r_m}} = \frac{5.5\,MPa\sqrt{m}}{\sqrt{0.796\times10^{-3}\,m}} = 194.5\,MPa$$

5.29 Two rods of Ni are broken in tension. One rod is nearly pure Ni whereas the other has been previously doped with Bi — a silver-colored, low melting point metal known to segregate to Ni grain boundaries. If SEM images of the fracture surfaces are compared, what micromechanisms and features should appear in the two cases?

The pure Ni is likely to be ductile, so it is likely to fracture in a fairly tough fashion. The plastic deformation would likely cause development of micro-voids on the fracture surface during the early stages of failure. These would appear as pockmarks on both fracture surfaces. The Bi-doped Ni is likely to brittle because of weak Bi segregating to the grain boundaries, so the likely fracture micromechanism is intergranular failure. The visible surface features would be the surfaces of the three-dimensional grains.

5.30 Two panes of ordinary float glass originally mounted in metal frames were broken in service. The panes were removed from the metal frames and photographs were brought to you for analysis. Pane (a) was damaged but intact; it was photographed to show one edge. Pane (b) was broken into multiple pieces; it was photographed to show a close-up of one fracture surface. The owner claims that both panes cracked as a result of mechanical overloading in the pane center in a direction perpendicular to the surface. First, assess the truth of this claim based on the visual evidence. Assuming that one or both truly could have fractured as claimed, determine on which side of the plate the pressure was applied, and in what direction the crack grew. Explain your rationale.

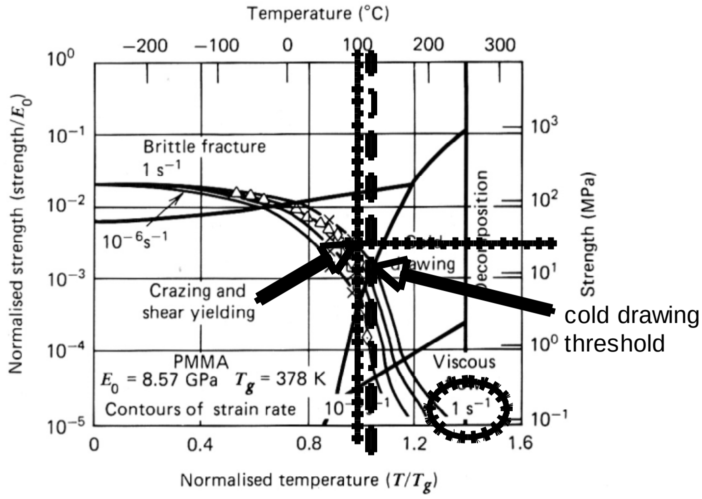
(a)  (b) 

[Photographs courtesy of Anthony Spizzirri]

It is unlikely that the failure of Pane (a) was caused by mechanical overload. The cracks began at the edge where it was connected to the metal frame, initially propagated approximately 90° to the edge, and then meandered about, terminating inside the glass. Together, these features imply that thermal fracture was the cause, perhaps because of differences in thermal expansion of the metal frame and the glass pane, or due to a temperature gradient. Pane (b) shows Wallner lines on the fracture surface. These are typical of mechanical failures, not thermal failures. Furthermore, the asymmetry of the lines indicates that the plate was bent with the tensile side on the top surface (in the orientation of the photo) and the crack origin to the right. These features are consistent with bending of a plate loaded somewhere in the center and constrained around the edges. The load would have been applied on the bottom of the plate in the orientation of the photo so the bottom would have been in compression and the top in tension.

5.31 A rod of PMMA is tested in tension at a strain rate of 1 s^{-1} to determine the maximum elastic stress it can bear. At a temperature of 100°C, what would you expect the approximate maximum elastic stress to be, and what mechanism would you expect to take over at that stress level: brittle fracture or plastic deformation by crazing/shear yielding? How do you know? If you want to induce plastic deformation by cold drawing instead, what should you change about the tensile test parameters?

Based on the Deformation Mechanism Map for PMMA, at a strain rate of 1 s^{-1} *and a temperature of 100°C the PMMA is likely to yield by crazing/shear yielding at approximately 20 MPa (dotted line — the stress axis is in log form, so some guesswork is involved). To induce cold drawing, the easiest option is to increase the temperature to at least 120°C (dashed line).*



Design

5.32 While preparing a SiC mirror for its role in an orbiting telescope, it becomes necessary for you to design a protective support system to ensure that the mirror does not fracture from the forces involved in boosting the satellite into orbit. For the purposes of this exercise, the mirror can be modeled as a flat disk 3.25 m in diameter and 3 mm thick. For evaluation, 20 small disks 100 mm in diameter x 3 mm thick prepared in the same fashion as the mirror are tested in bending. The results indicate that the characteristic strength is 474 MPa and the Weibull modulus is 16.9. What is the maximum bending stress that the full-scale mirror can be allowed to experience so that the probability of failure is no greater than 1%? No greater than 0.1%? Based on your results, would you recommend designing for the lower of the two failure probabilities even if it would increase the cost?

First we calculate the stress associated with a 1% failure probability for the test disks. We will assume that σ_u=0 because SiC is a brittle ceramic material. The specimen tested is considered the "unit volume" of interest, so the probability of failure is

$$F = 1 - \exp\left\{-\left(\frac{\sigma - \sigma_u}{\sigma_o}\right)^m\right\}$$ *so for our case, we need a maximum stress of*

$$\sigma_{1\%} = \sigma_0\left[-\ln(1-F)\right]^{\frac{1}{m}} = (474\,MPa)\left[-\ln(1-0.01)\right]^{\frac{1}{16.9}} = (474\,MPa)\left[0.0101\right]^{\frac{1}{16.9}}$$
$$\sigma_{1\%} = 361\,MPa$$

Then repeat for F = 0.1%=0.001

$$\sigma_{0.1\%} = \sigma_0\left[-\ln(1-F)\right]^{\frac{1}{m}} = (474\,MPa)\left[-\ln(1-0.001)\right]^{\frac{1}{16.9}} = (474\,MPa)\left[0.0010\right]^{\frac{1}{16.9}}$$
$$\sigma_{0.1\%} = 315\,MPa$$

Now we must scale the results to take into the account the fact that the mirror volume is much larger than the test specimen volume, so there is a higher probability of finding a large flaw.

The volume of a test disk is

$$V = t\pi\left(\frac{d}{2}\right)^2 = \pi(0.003m)\left(\frac{0.1m}{2}\right)^2 = 2.36\times10^{-5}\,m^3$$

The volume of the mirror is approximately

$$V = t\pi\left(\frac{d}{2}\right)^2 = \pi(0.003m)\left(\frac{3.25m}{2}\right)^2 = 0.0249\,m^3$$

Then the ratio of the stresses for an identical failure probability is given by

$\sigma_1/\sigma_2 = (V_2/V_1)^{1/m}$ *so that* $\sigma_1 = \sigma_2(V_2/V_1)^{1/m}$

For the 1% failure case the maximum stress in the mirror is therefore

$$\sigma_1 = \sigma_2(V_2/V_1)^{1/m} = (361\,MPa)\left(2.36\times10^{-5}\,m^3/0.0249\,m^3\right)^{1/16.9} = (361\,MPa)(0.662) = 239\,MPa$$

and for the 0.1% case it is

$$\sigma_1 = \sigma_2(V_2/V_1)^{1/m} = (315\,MPa)\left(2.36\times10^{-5}\,m^3/0.0249\,m^3\right)^{1/16.9} = (315\,MPa)(0.662) = 209\,MPa$$

As expected, the allowed stress is substantially lower for the large mirror than for the small test specimens. Given the relatively small difference in stress between the two cases, the small additional cost that is probably associated with achieving the additional 30 MPa stress decrease, and the enormous expense of the satellite, it is almost certainly worth designing the support structure for the lower failure probability.

5.33 In problem 26, the existing design of the notches will lead to local plastic deformation even though the applied load is elastic elsewhere in the bar. If the load, the material, and the dimensions *t*, *D*, *d*, and *h* cannot be altered, what is the minimum notch radius *r* that would prevent yielding (with no safety factor)?

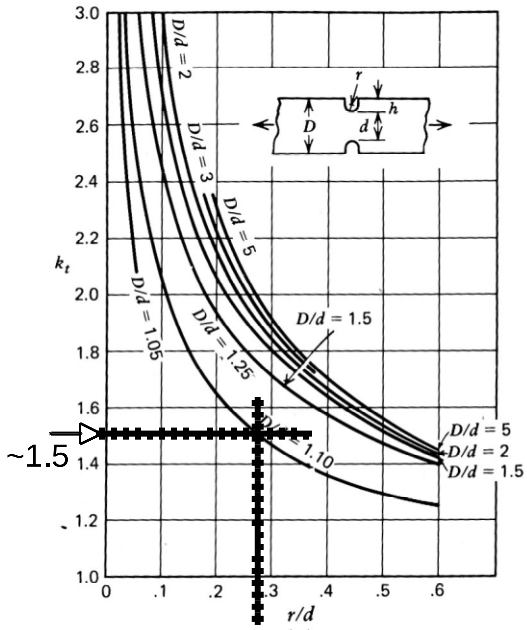
Begin by calculating the nominal stress on the bar:

$$\sigma_a = \frac{P}{A_0} = \frac{15\times10^3\,N}{(0.2\times10^{-2}\,m)(2\times10^{-2}\,m)} = 375\,MPa$$

Then determine the maximum stress concentration factor that can be tolerated to avoid a local stress greater than the yield strength of 950 MPa.

$$\sigma_{\max} = \sigma_a(1+2\sqrt{a/\rho}) = \sigma_a(1+k_t)$$
$$k_t = \frac{\sigma_{\max}}{\sigma_a} - 1 = \frac{950\,MPa}{375\,MPa} - 1 = 1.53$$

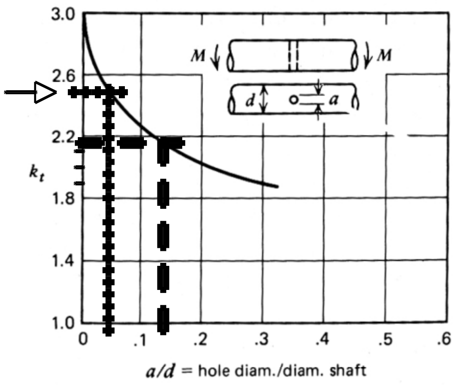
This leads to a minimum ratio of r/d~2.8 as shown using Fig. 5.9a. Because d is fixed at 1.8 cm, this requires that r = 5.04 cm. A pretty gentle notch in a bar 2 cm wide!



(a)

5.34 A long rod of solid 6061-T6 aluminum has a diameter of 2.54 cm. A hole 1.0 mm in diameter (intended for mounting a bracket to the rod) is drilled through the rod diameter as shown in Fig. 5.9(d). It is calculated that when the wind blows, the rod will be subjected to a nearly pure bending moment. It is found during testing that the rod is cracking at 90% of the design stress right at the hole. If the rod is reconfigured to safely reach the design stress, should the hole diameter be increased or reduced? What hole diameter range would meet the design criteria?

The rod dimensions are d=25.4 mm and a=1.0 mm. The ratio of a/d is needed to calculate k_t. In this case, a/d=1/25.4=0.04. On the plot in Fig. 5.6d, this corresponds to approximately k_t=2.5.



(d)

As seen from the curve, making the hole diameter larger will reduce the stress concentration factor (as expected from the 1/√ρ dependence of the basic k_t equation).

$$\sigma_{max} = \sigma_a(1+2\sqrt{a/\rho}) = \sigma_a(1+k_t)$$

The current applied stress at failure is 90% of the design stress. If we wish to increase the new applied stress at failure to a level exactly equal to the design stress, it must be

$$\sigma_a = 0.9\sigma_a'$$

The failure stress (i.e., the local maximum stress) is constant because it is a material parameter. Thus we can set the original condition and the desired condition equal to each other as follows:

$$\sigma_{max} = \sigma_a(1+k_t) = \sigma_a'(1+k_t')$$
$$0.9\sigma'(1+k_t) = \sigma_a'(1+k_t')$$
$$k_t' = 0.9(1+2.5) - 1 = 2.15$$

Referring back to the plot, we see that k_t=2.15 requires a/d=0.14. If the rod diameter is fixed at 25.4 mm, this means a=0.14(25.4mm)=3.6 mm. The hole should be redesigned for a diameter of 3.6 mm or slightly larger.

Extend

5.35 Acquire the National Transportation Safety Board (NTSB) report describing the 2007 failure of the I-35 bridge in Minneapolis. What was the "generally accepted practice among Federal and State transportation officials" that may have contributed to the disaster? If this practice had not been common, what evidence might have been gathered that would have prevented the disaster?

According to the NTSB report, "Contributing to the accident was the generally accepted practice among Federal and State transportation officials of giving inadequate attention to gusset plates during inspections for conditions of distortion, such as bowing, and of excluding gusset plates in load rating analyses." The report goes on to say that a 2003 study of the bridge included photographic evidence of gusset plate buckling, but documenting the *buckling was not the point of the photographs so no particular notice was taken. Other routine inspections also failed to comment on the buckling. Stress calculations performed after the failure showed that the buckling was evidence of underdesigned gussets. Had the buckling been noted in earlier inspections and this analysis been performed, it is possible that the underdesigned gusset plates would have been identified and the problem corrected.*

Reference: National Transportation Safety Board (2008). Collapse of I-35W Highway Bridge, Minneapolis, Minnesota, August 1, 2007. Highway Accident Report NTSB/HAR-08/03. Washington, DC.

5.36 In the National Transportation Safety Board (NTSB) report describing the 2007 failure of the I-35 bridge in Minneapolis it is reported that in October of 1998, bridge inspectors found 12 fatigue cracks in 8 girders. The largest of these was over 50 inches long. Acquire the NTSB report and explain what was done to limit further growth of these cracks. Did this work indefinitely?

After the discovery of the large fatigue cracks, "To prevent propagation of the largest crack, bridge maintenance workers initially drilled 2-inch-diameter holes at each end of the crack." This was intended to increase the radius at the fatigue crack tip, thereby reducing the stress concentration and significantly reducing the chance of fast fracture. This was deemed a temporary measure, and several additional steps were undertaken to further reduce the possibility of fracture (including enlarging the crack-arrest holes and bolting reinforcing plates to the cracked areas). Regular inspections of the cracked areas occurred thereafter. The fatigue cracks do not appear to have played a major role in the collapse of the bridge, so it would appear that the repair and inspection procedures were effective.

Reference: National Transportation Safety Board (2008). Collapse of I-35W Highway Bridge, Minneapolis, Minnesota, August 1, 2007. Highway Accident Report NTSB/HAR-08/03. Washington, DC.

5.37 Acquire a journal paper of your choosing that describes a failure analysis involving fractography. Reproduce at least one image of the fracture surface from the paper, and describe what feature(s) were important to the conclusions of the failure analysis. Provide a full citation for the paper in a standard reference format.

Answers will vary.

5.38 Find a journal paper that describes the effect of notch sensitivity on the fracture behavior of Ultra High Moleculra Weight Polyethylene (UHMWPE) components used in orthopedic implants. Summarize the findings of the paper, and provide a full citation for the paper in a standard reference format. Also explain why UHMWPE is chosen over other materials for the application described in the paper; be sure to include at least two design criteria and show how/why UHMWPE meets those criteria.

Answers may vary, but are likely to include the following paper or related papers:
M.C. Sobieraj, S.M. Kurtz, and C.M. Rimnac, "Notch strengthening and hardening behavior of conventional and highly crosslinked UHMWPE under applied tensile loading", Biomaterials 26(17), 3411-3426 (2005).

CHAPTER 6

Review

6.1 Summarize the fundamental concept behind Griffith's major contribution to understanding the fracture process.

Griffith recognized for the first time that the fracture process can be described as a balance between energies: the elastic energy stored in a material (an applied energy) and the surface energy associated with the new surfaces created during fracture (a material property).

6.2 Write Griffith's expression for the fracture stress of a material under plane stress conditions. Identify which parameters are *applied* and which are *material constants*.

The basic equation is $\sigma = \sqrt{\frac{2E\gamma_S}{{}^{1}a}}$ *. The stress and the crack length can be considered as applied parameters, and the Young's modulus and the surface energy are material constants.*

6.3 State the limitation of Griffith's analysis with regard to crack geometry, and explain why it is a limitation.

The Griffith equation does not include any explicit dimension for the crack tip radius. This is a limitation because the Griffith equation only applies to extremely sharp cracks. For cracks that are less sharp than the model assumes, the Griffith fracture criterion is only a lower bound for the fracture condition (i.e., the fracture stress will not be lower than the Griffith equation predicts, but it may very well be higher).

6.4 What is the initially-counterintuitive relationship between thermal shock resistance and microcrack density in a pre-cracked brittle material? Why, after some thought, does this relationship makes sense after all?

Thermal shock resistance is actually expected to increase with increasing microcrack density. Although it may appear surprising that a component with more cracks is more resistant to thermal crack growth, it makes sense once one realizes that all of the cracks are propagating by the same amount in response to the thermal shock, so higher crack density means more fracture surface area created for a given distance of crack propagation. This takes more energy, and is therefore less favorable, than an equivalent distance of crack propagation in the same material with lower initial crack density.

6.5 Summarize the two recommendations for choosing a thermal shock resistant material, making it clear under what condition(s) each recommendation applies.

When choosing a suitable brittle solid to maximize thermal shock resistance, it is first necessary to determine how severe the anticipated thermal shock will be (i.e., how large a temperature change is expected). For small temperature changes, preventing crack nucleation is the best approach. This is achieved by selecting a material with low thermal expansion and low elastic modulus, and high strength and high thermal conductivity. On the other hand, for large thermal changes optimal material properties should once again include low thermal expansion, but also low fracture strength and high elastic modulus. In addition, the material should ideally be designed to contain a high density of microcracks. Unfortunately, the latter recommendation limits the load-bearing capability of the material.

6.6 State the limitation of Griffith's analysis that led to the analyses of Orowan and Irwin, and explain why the additional work was necessary.

The Griffith equation was developed to describe the fracture of brittle materials like silicate glass. It does not include a term for any energy dissipation mechanisms other than surface work. This is a severe shortcoming with respect to metals and polymers that can undergo significant plasticity at the crack tip during the fracture process. Orowan and Irwin developed methods for including plasticity without losing the essence of the Griffith's energy-based approach.

6.7 Define the *strain (or elastic) energy release rate* first in words, then as an energy-based equation.

The strain (or elastic) energy release rate characterizes that portion of the total stored elastic energy in a body that is released when a crack propagates. The units are therefore energy per unit crack area. The basic equation that defines the term is

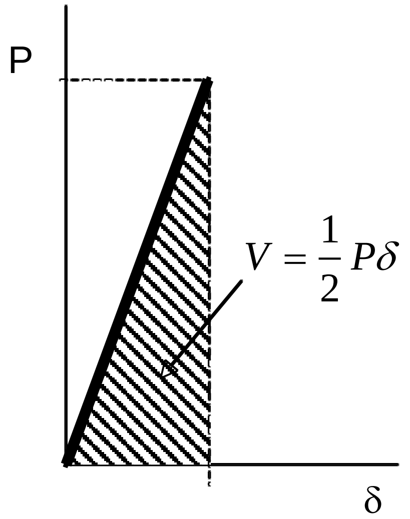
$G = 2(\gamma_S + \gamma_P)$, *where* ▌ *is the strain energy release rate, and* $\gamma_s + \gamma_P$ *are the surface energy and the plastic deformation energy, respectively, involved in the fracture process.*

6.8 What are the units of the *elastic strain energy release rate?*

Energy per unit area, just like the units of surface energy.

6.9 Sketch a load-displacement plot for a linear elastic material, then illustrate on the plot how V (the stored strain energy) is represented in the sketch.

The absolute stored strain energy is the area under the elastic curve, just like the elastic strain energy density is the area under the elastic portion of a stress-strain curve.



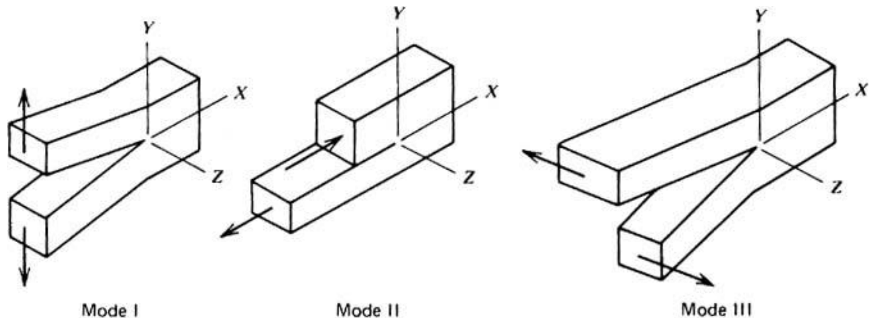
6.10 As a crack advances, what happens to the stiffness of the cracked body? What happens to the compliance?

The overall stiffness of a cracked body is reduced as a crack advances because there is less material (fewer bonds) supporting the load. The compliance is just the inverse of the stiffness, so the compliance increases as the crack advances.

6.11 What important information can be learned from a Charpy impact test that can be used for design purposes, particularly with steels? What phenomenon limits the direct applicability of the transition temperature derived from standardized Charpy impact test results to the design of many structures?

The energy of fracture can be measured at different temperatures using the Charpy test. From this, a transition temperature between low energy and high energy fracture can be determined for materials (like ferritic steels) that exhibit a ductile to brittle transition. This information can be used for design purposes to determine the minimum temperature below which the material should not be used if fracture is to be avoided. However, the transition temperature is a function of specimen size, so a component that does not match a standard Charpy specimen in thickness is likely to undergo the brittle to ductile transition at a different temperature. This is problematic if it causes either an unsafe design (the transition temperature is higher than expected and the component is underdesigned) or an inefficient design (the transition temperature is lower than expect and the component is overdesigned).

6.12 Sketch the Mode I, Mode II, and Mode III crack opening geometries, and clearly state which is the most commonly found in engineering structures.



Of these, Mode I (or a large component of Mode I) is the most common in real structures.

6.13 Define *plane stress* and *plane strain*, making clear which, if any, of the stresses are zero in each case.

Plane stress involves loading in which all of the stress exists only in a single plane, e.g., the X-Y plane. Thus $\sigma_z \approx 0$ *for this case. Plane strain loading has triaxial stresses even though loading may only be along X or Y, and strains only in one plane (again the X-Y plane). Because no strain is allowed along the Z axis, the stress along that axis is related to the X and Y stresses through the Poisson contraction:* $\sigma_z \approx \nu(\sigma_x + \sigma_y)$.

6.14 Make a table that clearly shows the constant geometric Y values that apply to very short cracks that are (i) center through-cracks, (ii) edge through-cracks, (iii) circular embedded cracks, and (iv) semicircular edge cracks.

	Y' factor	*Y factor*
center through-cracks	*1.0*	$1.0\sqrt{\pi}$
edge through-cracks	*1.12*	$1.12\sqrt{\pi}$
circular embedded cracks	$2/\pi$	$(2/\pi)\sqrt{\pi}$
semicircular edge cracks	$1.12(2/\pi)$	$1.12(2/\pi)\sqrt{\pi}$

6.15 Name and define K, K_C, and K_{IC}. Explain the differences and the conditions under which each parameter applies. State the units for each parameter.

K is the stress intensity factor. It includes a stress contribution, so it can have any value. K_C *is the critical stress intensity factor, of the fracture toughness. It is a specific value for a given set of conditions because it represents the critical stress intensity to cause crack growth.* K_{IC} *is the Mode I plane strain fracture toughness. It is a single critical value determined under a Mode I test configuration that leads to a plane strain condition at the crack front. The units for all three parameters are the same:* $MPa\sqrt{m}$.

6.16 Starting with Eq. 6-42, derive the expression for the plastic zone radius. Why is the cross section of the plastic zone not really a circle?

$$\sigma_{ys} = \frac{K}{\sqrt{2^1\ r}} \text{ so } \sqrt{2^1\ r} = \frac{K}{\sigma_{ys}} \text{ and } 2^1\ r = \frac{K^2}{\sigma_{ys}^2} \text{ so finally } r = \frac{K^2}{2\pi\sigma_{ys}^2}$$

The plastic zone is not a simple circle but rather dumbbell-shaped to account for both plane stress zones at the surfaces and plane strain at the sample interior.

6.17 What must be true about the dimensions of a plate in order for plane strain conditions to apply?

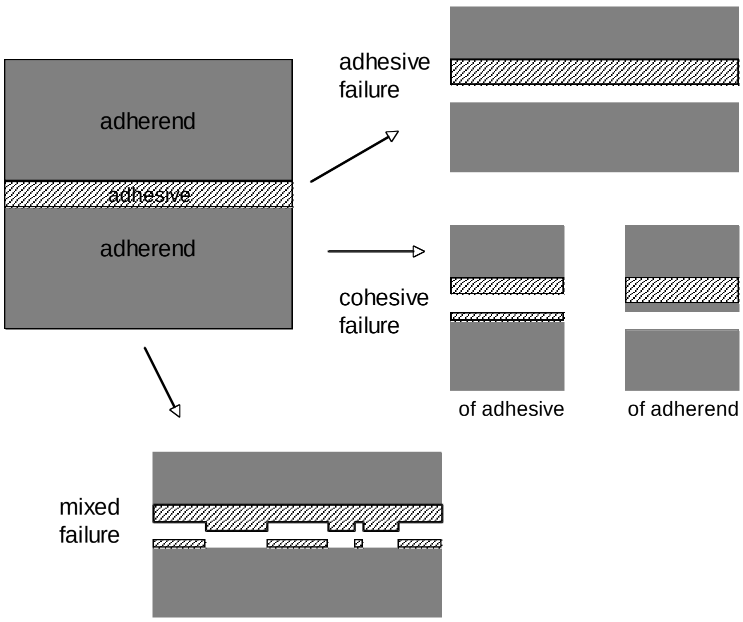
The plate must be thick relative to the plastic zone size so that r_y/t << $^1/_{10}$. Alternately, the plate thickness and crack size must be

$$t \text{ and } a \text{ Ö } 2.5\left(\frac{K_{IC}}{\sigma_{ys}}\right)^2$$

6.18 What can be learned by comparing the relative size of the shear lips and the flat surface of a metal failure?

Shear lips indicate plasticity involved in the fracture process, associated with plane stress conditions. If there is a large fraction of the fracture surface that is slanted with respect to the tensile axis, there was a large degree of plasticity involved in the fracture, and a large degree of plane stress condition relative to that of plane strain. The failure was probably a tough failure. If, however, the fraction of the fracture surface that is slanted is small, there was little plasticity, predominantly plane strain conditions, and the failure was probably fairly brittle.

6.19 Sketch an adhesive joint, labeling the adherends and the adhesive. Then sketch the same joint as it would look if it had been damaged in (a) an adhesive failure, (b) a cohesive failure, and (c) a mixed mode of failure.



Practice

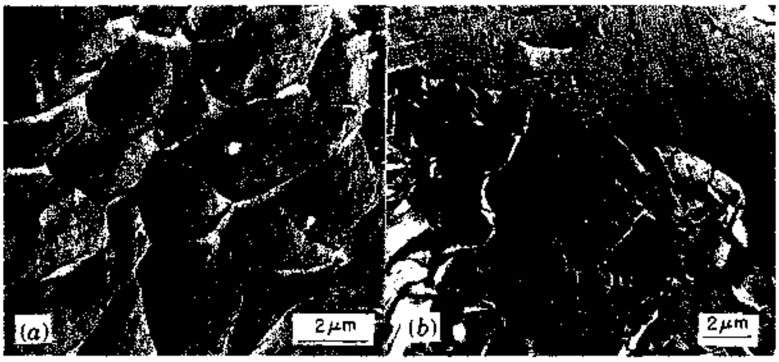
6.20 Use the Griffith analysis to calculate the critical stress required for the propagation of an edge crack of length 0.05 mm in a thin plate of soda-lime glass, assuming that the specific surface energy of this glass is 0.30 J/m^2.

Assuming that a plane stress condition applies, $\sigma = \sqrt{\frac{2E\gamma_S}{{}^1 a}}$.

For soda-lime glass, γ_S*=0.3 J/m*^{2} *and* $E \approx 69$ *GPa. The crack length 'a' in this case is the full crack length — 0.05 mm, or 5x10*$^{-5}$ *m — because it is an edge-cracked plate. The minimum critical stress is therefore*

$$\sigma_c = \sqrt{\frac{2E\gamma_S}{{}^1 a}} = \sqrt{\frac{2\left(69\times10^9\, N/m^2\right)\left(0.3 N\cdot m/m^2\right)}{{}^1\left(5\times10^{-5}\, m\right)}} = 16.2\ MPa$$

6.21 Two sets of Charpy specimens from two different materials (*X* and *Y*) and of different sizes were tested over a wide temperature range. Unfortunately, the pictures were not labeled, so we do not know from which samples they were obtained.



(a) If photographs *a* and *b* were both taken from Material *X*, speculate on the type of material that was tested and what the relative temperatures were corresponding to photographs *a* and *b*.

Material X could well have been a ferritic steel, with photograph 'b' corresponding to a lower test temperature than that associated with photograph 'a'.

(b) If photograph *a* is representative of the fracture surface in Material *Y* at all test temperatures, speculate on the identity of Material *Y*.

This material is most likely a non-ferrous alloy (such as a high strength Al alloy).

(c) Given your answers to the previous questions, speculate on the reason or reasons why Materials *X* and *Y* both showed a tough-to-brittle transition in the specimen sets tested.

Material X showed a tough-to-brittle transition as a result of a micromechanism transition from microvoid coalescence to cleavage and a plane stress-plane strain transition. Material Y experienced *only the plane stress-plane strain transition, so the toughness changed but the fracture micromechanism remained the same.*

(d) Name the fracture mechanisms shown in photographs *a* and *b,* and describe the process for their formation.

Photograph A: microvoid coalescence involving particle rupture or debonding followed by void growth and eventual coalescence. Photograph B: cleavage involving crack propagation along specific crystallographic planes.

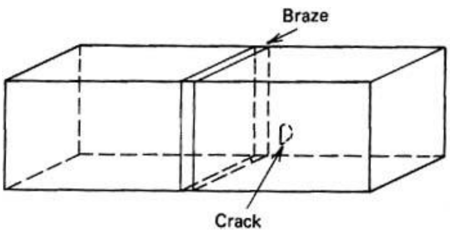
6.22 Consider a steel plate with a through-thickness edge crack like the one shown in Fig. 6.21f. The plate width (*W*) is 75 mm, and its thickness (*t*) is 12.0 mm. Furthermore, the plane strain fracture toughness and yield strength values for this material are 80 MPa $m^{1/2}$ and 1200 MPa, respectively. If the application in which the plate is used is expected to cause a stress of 300 MPa along the axis perpendicular to the crack, would you expect failure to occur if the crack length *a* is 15 mm? Explain.

The equation in Fig. 6.21f is written in terms of the plate dimensions, where P/tW is just the applied stress (which is already given to us). In the form of the equation that applies to this graph, the Y value includes the $\sqrt{\pi}$ term. To determine Y, the ratio of a/W must be calculated. a/W = 15mm/75mm = 0.2, so the graph indicates that Y≈2.5. Finally

$$K = Y\frac{P}{tW}\sqrt{a} = Y\sigma_{applied}\sqrt{a} = (2.5)(300\ MPa)(0.015\,m)^{0.5} = 91.9\ MPa\sqrt{m}$$

Since $K_I > K_{IC}$ under these conditions, failure is expected to occur.

6.23 Two square steel rods were brazed end-to-end to produce a rod with dimensions 6.25 × 6.25 × 30-cm. The silver braze is 0.063-cm thick and was produced with material possessing an ultimate strength of 140 MPa in bulk form. The steel rod sections have yield and tensile strengths of 690 and 825 MPa, respectively, and a plane-strain fracture-toughness value of 83 $MPa\sqrt{m}$.



(a) If the rod/braze assembly is loaded in tension perpendicular to the joint plane, will failure occur in the braze joint or in the steel? Assume that the steel rod contains an elliptical surface flaw 1.25-cm deep and 3.75-cm wide that is oriented normal to the stress axis. Determine the stress necessary for failure.

The braze t/w ratio is approximately 0.01. There will be maximum constraint on the braze with braze failure occurring when a stress 2.5-3 times the braze strength is applied (i.e. 350MPa-420MPa).

For an elliptical crack (a=1.25cm, 2c=3.75cm), a/2c = 0.33. Assuming $\sigma/\sigma_{ys} \approx 0.5, Q \approx 1.65$ *(Fig. 6.21h)*

$$K = 1.1\sigma\sqrt{\pi a}\,; 83 = 1.1\sigma\sqrt{\frac{\pi(1.25 \times 10^{-2})}{1.65}}$$

$$\sigma = 489\ MPa$$

In this instance, the stress needed to propagate the crack is greater than that needed to exceed the braze strength, so the braze joint would fail first.

(b) If the same rod had instead contained a through-thickness crack of depth 2.5-cm at the same crack location and orientation, determine where failure will occur and at what stress level.

For an edge crack = 2.5 cm and w = 6.25 cm, from Fig. 86.21f, $Y \approx 3.8$

$$K = Y\sigma\sqrt{a}$$

$$83 = 3.8\sigma\sqrt{2.5 \times 10^{-2}}$$

$$\sigma = 138\ MPa$$

In this instance, the steel would crack even though the applied stress is approximately the yield stress of the braze material tested on its own. The braze is actually stronger because of the constraint imposed by the steel blocks.

6.24 Cortical bone is decidedly anisotropic in its fracture behavior. Crack propagation parallel to the long axis of a bone is much easier than perpendicular to the axis (i.e., “across” the bone). When fracture of a human tibia is measured in the parallel crack configuration, an average value of K_{IC} = 3.95 MPa-m$^{1/2}$ is found. Imagine that a hole is drilled through cortical bone in a direction perpendicular to the bone axis. The hole is intended to be used for insertion of a pin associated with repair of a nearby fracture. Imagine that in the hole drilling process, a *semi-circular* crack is created emanating from the drilled hole aligned with the osteon cement lines (see Figure 6.22*c* for clarification). Assuming that the cortical bone is 3 mm thick, the modulus can be estimated as E = 20 GPa, and that during the surgery the worst-case tension strain that the bone will experience perpendicular to the crack is 2x10^{-3}, what is the maximum tolerable crack radius to avoid fracture at the drilled hole?

A crack is most likely to develop along the weak interfaces parallel to the long axis of the bone, as described here. Note that a large hoop stress would be an unlikely scenario for a long bone (mostly loaded in tension, compression, or bending) but we can evaluate the possibility nonetheless. The goal is to find the critical crack length a_c.

$$K_A \approx 1.12 \cdot \sqrt{1/Q} \cdot \sqrt{\sec\frac{{}^{1}a}{2t}} \cdot 3\sigma\sqrt{{}^{1}a}$$

$\sigma_{\max} = E\varepsilon_{\max} = 40\,MPa$

$\sqrt{1/Q} = 2/\pi$ *because the crack is exactly semicircular.*

$$3.95\,MPa\sqrt{m} \approx 1.12 \cdot \frac{2}{\pi} \cdot \sqrt{\sec\frac{{}^{1}a}{2(0.003m)}} \cdot 3(40\,MPa)\sqrt{{}^{1}a}$$

In this case a_c cannot be determined explicitly, so a numerical solution using a tool like Excel's Goal Seek function must be used. For this scenario, a_c=0.68 mm. It is a good thing that significant circumferential tensile loading of bone is not likely to occur!

6.25 A thin-walled pressure vessel 1.25-cm thick originally contained a small semicircular flaw (radius 0.25-cm) located at the inner surface and oriented normal to the hoop stress direction. Repeated pressure cycling enabled the crack to grow larger. If the fracture toughness of the material is $88\,\mathrm{MPa}\sqrt{\mathrm{m}}$, the yield strength equal to 825 MPa, and the hoop stress equal to 275 MPa, would the vessel leak before it ruptured?

The basic equation is $K = Y\sigma\sqrt{a}$ *.*

For leak-before-break conditions, critical crack length a_c > t, the wall thickness. There are several equivalent approaches that may be used. The wall thickness can be substituted for crack length to compute K, which can then be compared to the fracture toughness. Or, the fracture toughness can be entered into the equation so that the critical crack length at fracture can be calculated for comparison to the wall thickness. This is the approach used here, but either will do.

$$a_c = \frac{1}{\pi}\left(\frac{K_c}{Y\sigma}\right)^2 = \frac{1}{\pi}\left(\frac{88\,MPa\sqrt{m}}{(2/\pi)(1.12)(275\,MPa)}\right)^2 = 0.064\,m = 6.6\,cm$$

Because 6.6 cm > 1.25 cm, the crack will reach the outer surface of the wall before the crack length reaches the critical value. The vessel will therefore leak before it breaks.

6.26 A 3 cm-diameter penny-shaped slag inclusion is found on the fracture surface of a very large component made of steel alloyed with Ni, Mo, and V. Could this defect have been responsible for the fracture if the stress acting on the component was 350 MPa? The only material data available are Charpy results in the transition temperature regime where impact energy values of 7-10 ft-lb were reported. Be careful of units when you consult the K_{IC}–CVN conversions.

From Table 6.5, we can see that the K_{IC}–CVN conversions require a yield strength, which we do not have. Instead, we can use either the Barsom-Rolfe conversion or the Sailors-Corten conversion, and make a reasonable guess as to the Young's modulus which doesn't vary much from one steel alloy to another. From Failure Analysis Case Study 6.3,

$$\frac{K_{IC}^2}{E} = 2(CVN)^{3/2} \quad \text{(Barsom-Rolfe)}$$ *which gives units of $psi\sqrt{in}$ when K_{IC} and E are in $psi\sqrt{in}$ and psi, and CVN is in ft-lb units.*

So, for CVN = 7 ft-lb

$$K_{IC} = \sqrt{2E(CVN)^{3/2}} = \sqrt{2\left(30\times10^6\ psi\right)\left(7\ ft-lb\right)^{3/2}} = 33335\ psi\sqrt{in} = 33.3\,ksi\sqrt{in}$$
$$K_{IC} = 33.3\,ksi\sqrt{in}\left(1.099\ MPa\sqrt{m}\ /\ ksi\sqrt{in}\right) = 36.6\ MPa\sqrt{m}$$

and for CVN = 10 ft-lb

$$K_{IC} = \sqrt{2E(CVN)^{3/2}} = \sqrt{2\left(30\times10^6\ psi\right)\left(10\ ft-lb\right)^{3/2}} = 43559\ psi\sqrt{in} = 43.6\,ksi\sqrt{in}$$
$$K_{IC} = 43.6\,ksi\sqrt{in}\left(1.099\ MPa\sqrt{m}\ /\ ksi\sqrt{in}\right) = 47.9\ MPa\sqrt{m}$$

Now we determine the critical stress intensity factor for the failure under the conditions described, assuming that the embedded circular crack is responsible. We will also assume that there is no need for a finite size correction since the inclusion was found in "a very large component". The stress intensity factor for the penny-shaped embedded crack is therefore

$$K_{failure} = Y\sigma\sqrt{\pi a} = \left(\frac{2}{\pi}\right)\left(350\ MPa\right)\sqrt{\pi\left(0.015m\right)} = 48.4\ MPa\sqrt{m}$$

Since this value is greater than the highest fracture toughness estimate based on the K_{IC}–CVN conversions, it is likely that this slag inclusion was responsible for fracture of the steel component.

6.27 A compact tension test specimen ($H/W = 0.6$), is designed and tested according to the ASTM E399-90 procedure. Accordingly, a Type I load versus displacement (P vs. δ) test record was obtained and a measure of the maximum load P_{max} and a critical load measurement point P_Q were determined. The specimen dimensions were determined as W = 10-cm, t = 5-cm, a = 5-cm, the critical load-point measurement point P_Q = 100 kN and P_{max} = 105 kN. Assuming that all other E399 requirements regarding the establishment and sharpness of the fatigue starter crack are met, determine the critical value of stress intensity. Does it meet conditions for a valid K_{IC} test if the material yield stress is 700 MPa? If it is 350 MPa?

Since $P_{max}/P_Q < 1.1$, we may compute K_Q.

$$K_Q = \frac{Y\left(P\sqrt{a}\right)}{tw}\ ;\ From\ Fig. 8.7g\ for\ \frac{a}{w} = 0.5, Y = 13.5$$

$$K_Q = \frac{13.5(100000)\sqrt{0.05}}{(0.05)(0.1)} = 60.4\ MPa\sqrt{m}$$

Is $K_Q = K_{IC}$?

$$a\ \&\ t\ \geq 2.5\left(\frac{K_Q}{\sigma_{ys}}\right)^2$$

$When\ \sigma_{ys} = 700\ MPa, a\ \&\ t\ \geq 1.86\ cm. Therefore, K_Q = K_{IC}\ since\ a = t = 5\ cm$

$When\ \sigma_{ys} = 350\ MPa, a\ \&\ t\ \geq 7.4\ cm. Therefore, K_Q \neq K_{IC}$

$since\ a = t = 5\ cm\ is\ less\ than\ required\ 7.4\ cm.$

6.28 An infinitely large sheet is subjected to a gross stress of 350 MPa. There is a central crack 5/π-cm long and the material has a yield strength of 500 MPa.

(a) Calculate the stress-intensity factor at the tip of the crack.
(b) Calculate the plastic–zone size at the crack tip.
(c) Comment upon the validity of this plastic–zone correction factor for the above case.

$\sigma = 350\ MPa; 2a = \frac{5}{\pi}\ ;\ \sigma_{ys} = 500\ MPa$

$a)\ K = \sigma\sqrt{\pi a} = 350\sqrt{\frac{\pi(2.5\times 10^{-2})}{\pi}}\ ;\ K = 55.3\ MPa\sqrt{m}$

$b)\ r_y = \frac{1}{6\pi}\left(\frac{K^2}{(\sigma_{ys})^2}\right) = 0.65\ mm\ (plane\ strain) = 1.95\ mm\ (plane\ stress)$

c) $K_{eff} = \frac{\sigma\sqrt{\pi a}}{\left[1-\frac{1}{2}\left(\frac{\sigma}{\sigma_{ys}}\right)^2\right]} = 63.6\ MPa\sqrt{m}$

$\frac{K_{eff}}{K_{elastic}} = 1.15. This\ 15\%\ difference\ is\ on\ the\ border\ line\ of\ an\ acceptable\ correction$

6.29 Is it possible to conduct a valid plane strain fracture toughness test for a CrMoV steel alloy under the following conditions: $K_{IC} = 53\text{MPa}\sqrt{\text{m}}$, σ_{ys} = 620 MPa, W = 6 cm and plate thickness, B = 2.5 cm?

Fracture toughness tests utilize a/w = 0.5. Therefore, a = 3 cm. For a valid K_{IC} test,

$a\ \&\ B\ \geq\ 2.5\left(\frac{K_{IC}}{\sigma_{ys}}\right)^2 \geq 1.83\ cm$

Since a = 3 cm and B = 2.5 cm for this sample, a valid plane strain fracture toughness test could be conducted under these conditions.

6.30 If the plate thickness in the previous problem were 1 cm, would the thickness be sufficient for a J_{IC} test? Assume these material properties: E = 205 GPa, ν = 0.25.

Since a valid plane strain fracture toughness test requires that

$$a \,\&\, B \geq 2.5\left(\frac{K_{IC}}{\sigma_{ys}}\right)^2 \geq 1.83\ cm$$

We see that a valid K_{IC} test cannot be conducted. On the other hand, a valid J_{IC} test requires that specimen thickness, B, and unbroken ligament, b_0.

$$b_0 = W - a \geq 25\left(\frac{J_Q}{\sigma_{ys}}\right)$$

$$J_{IC} = \left(\frac{K_{IC}^2}{E}\right)(1-\nu^2) = 12846\ N/m$$

$$B\&b_0 \geq \frac{25(12846)}{620 \times 10^6} = 0.52\ mm$$

Therefore, the specimen dimensions would be satisfactory for a valid J_{IC} test. Note how much smaller the minimum necessary specimen dimensions are for a J_{IC} test than for a K_{IC} test.

6.31 A rod of soda-lime-silica glass is rigidly constrained at 400°K and then cooled rapidly to 300°K. Assume that E=70 GPa, σ_{ts}=90 MPa, $\alpha=8\times10^{-6}K^{-1}$, and $K_{IC} = 0.8\,\text{MPa}\sqrt{\text{m}}$.

(a) With no visible surface damage, could you expect the rod to survive this quench?

Begin by calculating the thermal stress.

$$\sigma = \alpha E \Delta T = (8\times10^{-6})(70\times10^9)(100) = 56\ MPa$$

Since $\sigma_{thermal} < \sigma_{TS}$, the rod would probably survive (this assumes that the condition of the rod is similar to the condition of the rod originally used to determine the tensile strength).

(b) If the glass rod contained a 1-mm scratch that was oriented perpendicular to the axis of the rod, would your answer to part (a) be the same?

$$K = \sigma\sqrt{\pi a} = (56\times10^6)\sqrt{\pi(10^{-3})} = 3.14\ MPa\sqrt{m}$$

$$K > K_{IC}\ ; failure$$

Alternatively, the problem could be solved by calculating the allowable stress for the given fracture toughness

$$0.8\times10^6 = \sigma\sqrt{\pi(0.001)}$$

$$\sigma = 14.3\ MPa, which\ is\ less\ than\ thermal\ stress; therefore, failure\ will\ occur.$$

(c) Would failure occur if the temperature drop and the crack size were each half the values given above?

The stress changes linearly with temperature. The stress intensity factor is non-linear with crack length. Clearly the situation is less severe than in part (b), but how much less severe?

$$K = \sigma\sqrt{\pi a} = (28 \times 10^6)\sqrt{\pi(0.5 \times 10^{-3})} = 1.10\ MPa\sqrt{m}$$

$$K > K_{IC}\ ; failure$$

Alternatively,

$$0.8 \times 10^6 = \sigma\sqrt{\pi(0.0005)}$$

$$\sigma = 20.2\ MPa, which\ is\ less\ than\ thermal\ stress\ and\ failure\ will\ still\ occur.$$

6.32 A set of double cantilever beam adhesion test specimens was fabricated with 6061-T6 aluminum alloy beams. Each beam had dimensions of 76.2 x 12.7 x 12.7 mm. A bondline approximately 250 micrometers thick was created using either polyimide A or polyimide B, and a precrack of identical length was formed in each specimen.

(a) If the average $\mathcal{G}_{Ic}$ values for A and B were 19 and 63 J/m^2, respectively, what was the ratio of the critical loads?

The equation relating the critical strain energy release rate to DCB fracture conditions is

$$\mathcal{G}_{IC} = \frac{12P^2a^2}{b^2h^3E_s}$$

The dimensions and adherend materials are constant, so

$$\mathcal{G}_{IC} \propto P^2$$

$$\frac{\mathcal{G}_{IC,A}}{\mathcal{G}_{IC,B}} = \frac{P_A^2}{P_B^2}$$

$$\frac{P_A}{P_B} = \sqrt{\frac{\mathcal{G}_{IC,A}}{\mathcal{G}_{IC,B}}} = \sqrt{\frac{19}{63}} = 0.55$$

In other words, the load to fail polyimide A was approximately half that required to fail polyimide B.

(b) If the DCB specimen length was doubled, how would the critical load be expected to change? Explain.

Doubling the length would make no difference to the critical load. As long as the initial crack length is small compared to the beam length, simple beam theory applies and the stress intensity at the crack tip is independent of beam length. This is similar to the constant Y factor that applies to cracked plates when crack size to plate width ratio, a/W, is very small.

Design

6.33 For the Ti-6A1-4V alloy test results given in Table 8.2, determine the sizes of the largest elliptical surface flaws ($a/2c$ = 0.2) that would be stable when the design stress is 75% of σ_{ys}.

For Ti-4Al-4V, the necessary values are available from Table 6.3

	$K_{IC}\,(MPa\sqrt{m})$	$\sigma_{ys}\ (MPa)$	$\sigma_{design}\,(MPa)$
1	115.4	910	682.5
2	55.0	1035	776.25

*From Fig. 6.21h with a/2c=0.2, Q=1.22*$a/2c = 0.2 \varphi = 1.22$

$$K = Y\sigma\sqrt{\frac{\pi a}{Q}}$$

Ignoring for the moment the Y factor for the surface flaw,
For alloy 1: a_{max} *= 11.14 mm (and* c_{max} *= 27.86 mm)*

For alloy 2: a_{max} *= 1.96 mm (and* c_{max} *= 4.89 mm)*

If one adds a surface flaw correction of Y=1.12, then critical crack length a_c *would decrease to approximately 9.0 and 1.6 mm, respectively for the two Ti alloys.*

6.34 A plate of 4335 + V steel contains a semi-elliptical surface flaw, 0.8-mm deep and 2-mm long, that is oriented perpendicular to the design stress direction. Given that $K_{IC} = 72.5\,\text{MPa}\sqrt{\text{m}}$, σ_{ys} = 1340 MPa, and the operating stress = 0.4σ_{ys}, determine whether the plate is safe for continued service, based on the requirement that the operative stress intensity level is below 0.5 K_{IC}.

Since the plate contains a semi-elliptical surface flaw, the stress intensity factor is

$$K = 1.1\sigma_d\sqrt{\frac{\pi a}{Q}}$$

The aspect ratio of the crack is $a/2c = 0.8/2 = 0.4$

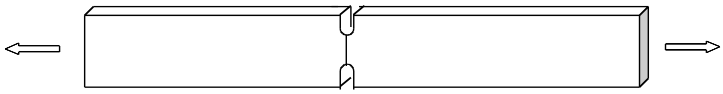
From Fig. 6.21h, the flaw shape parameter, Q, is estimated to be 1.95

$$K = 1.1(0.4)(1340)\sqrt{\frac{\pi(0.0008)}{1.95}}$$

$$K = 21.2\ MPa\sqrt{m}$$

Since the operative K level of 21.2 MPa$\sqrt{m}$ *is less than the design limit of 0.5*K_{IC} *(i.e. 36.25 MPa*$\sqrt{m}$*), the plate is safe for continued service.*

6.35 A research group is beginning a new project to measure the fracture toughness of individual grain boundaries in Cu-Bi alloys. They plan to make tiny specimens that have one grain boundary across the center line, as shown (not to scale). The center section will be notched to ensure that the fracture occurs at the grain boundary. It will be tested in tension along its length. The initial design for the specimen calls for dimensions 100 µm long, 20 µm wide, and 0.5 µm thick. The notches will each be 3 µm long x 0.25 µm wide. The radius of curvature at the tip of each notch will be 0.125 µm. The elastic modulus of pure Cu is 110 GPa (assume isotropic behavior), and it has a yield strength of approximately 70 MPa. The fracture toughness previously measured for a Bi-embrittled grain boundary in Cu was around 2 MPa√m.



(a) Why are the researchers so confident that this sample *shape* will cause fracture at the grain boundary and not somewhere else? Please give a brief answer.

First, the reduced cross-section will cause higher stress in the grain boundary region. Second, the notches will cause stress concentrations right at the boundary.

(b) Is this test likely to produce a K_{IC} value, or just a "generic" K_C value?

One of the criteria for plane strain is related to specimen thickness:

$$t \geq 2.5\left(\frac{K_{IC}}{\sigma_y}\right)^2 = 2.5\left(\frac{2\,MPa\sqrt{m}}{70\,MPa}\right)^2 = 2.1\times10^{-3}\,m = 2.1mm$$

Since the proposed specimen thickness is 0.5 µm, it is nowhere close to giving a plane strain fracture toughness value.

(c) The researchers need to choose a load cell with an appropriate range for this test. Would a load cell with a maximum force of 10 mN be sufficient to measure the load needed to fracture the doped boundary?

We will assume that the crack behavior is close to that of a "sharp" crack so Fig. 6.21 can be used to determine the Y factor. In this case, 2a/W=6µm/20µm=0.3. The chart gives a Y value of approximately 2.

$$K_c = Y\left(\frac{P}{A}\right)\sqrt{a} \text{ so}$$

$$P = \frac{K_c A}{Y\sqrt{a}} = \frac{\left(2\times10^6\,Pa\sqrt{m}\right)\left(20\times10^{-6}\,m\right)\left(0.5\times10^{-6}\,m\right)}{2\sqrt{3\times10^{-6}\,m}} = 5.8\times10^{-3}\,N = 5.8\,mN$$

It looks likely that a 10 mN load cell will be adequate.

(d) Is it likely that the Cu will behave elastically up to the point of fracture?

If the loading is elastic right up to the point of fracture, the local stress can be estimated using the stress concentration factor:

$$\sigma_{local} = \sigma_{applied}\left(1+2\sqrt{\frac{a}{\rho}}\right) = \left(\frac{P}{A}\right)\left(1+2\sqrt{\frac{a}{\rho}}\right)$$

$$\sigma_{local} = \left(\frac{5.8\times10^{-3}\,N}{(20\times10^{-6}\,m)(0.5\times10^{-6}\,m)}\right)\left(1+2\sqrt{\frac{3\mu m}{0.125\mu m}}\right) = 6.26\,GPa$$

This is clearly much greater than the yield strength of the Cu, so plasticity prior to fracture is likely.

6.36 An unreinforced polymeric pressure vessel is constructed with a diameter d = 0.44 m and a length L = 2 m. The vessel is designed to be capable of withstanding an internal pressure of P = 7 MPa at a nominal hoop stress of 70 MPa. However, in service the vessel bursts at an internal pressure of only 3.5 MPa, and a failure investigation reveals that the fracture was initiated by a manufacturing-induced semicircular internal crack 2.5 mm in radius.

(a) Based on the original design criteria, what is the wall thickness of this pressure vessel? You may assume that it is a thin-walled vessel for this calculation, even though this may not be true.

Using the thin-walled pressure vessel equation from Chapter 1:

$$\sigma_h = \frac{PR}{t}$$

$$t = \frac{PR}{\sigma_h} = \frac{(7\,MPa)(0.22\,m)}{17.5\,MPa} = 0.022\,m = 22\,mm$$

(b) Calculate the fracture toughness (K_{Ic}) of the material used.

At fracture, P=3.5 MPa so the hoop stress at the time must have been

$$\sigma_h = \frac{PR}{t} = \frac{(3.5\,MPa)(0.22\,m)}{0.022\,m} = 35\,MPa$$

The semicircular surface crack has a geometric factor Y=(1.12)(2/π) so that

$$K_{IC} = Y\sigma\sqrt{\pi a} = (1.12)\left(\frac{2}{\pi}\right)(35\,MPa)\sqrt{\pi(0.0025\,m)} = 2.21\,MPa\sqrt{m}$$

(c) Given the following materials to choose among, is it possible for this pressure vessel to meet a leak-before-break criterion at the original design stress without reinforcing the polymer or changing the vessel dimensions?

	PMMA	PS	PC	PET	PVC	PP

K_{IC} (MPa√m)	1.65	1.1	3.2	5.0	3.8	4.3

We want $a_c>t$ at the design stress of 70 MPa. An efficient approach is to set the crack length equal to the wall thickness in order to determine the fracture toughness at which $a_c=t$. This would be the minimum fracture toughness to establish a leak-before-break condition.

$$K_{IC} = Y\sigma\sqrt{\pi t} = (1.12)\left(\frac{2}{\pi}\right)(70\,MPa)\sqrt{\pi(0.0022\,m)} = 4.15\,MPa\sqrt{m}$$

From the list of materials given in the table, the available PET (polyethylene terephthalate) and PP (polypropylene) materials would both do the job.

6.37 Table 1.12 lists thermal characteristics for several ceramic materials at room temperature, along with a thermal shock resistance parameter *R'*. Rank order the materials from highest shock resistance to lowest, assuming that each is in pristine condition, and is cooled very rapidly from 500 °C to 100 °C. If the materials are slightly damaged but are exposed only to thermal fluctuations of 100 °C, re-rank the likely order (you may assume that the surface energy is the same for all). For a certain application you may need to design with a single material to survive both scenarios. If you may select only one material for further investigation, and your decision must be made entirely on the basis of shock resistance, which material from the lists would you select?

For severe shock of material with few flaws, resistance for crack initiation is expressed by

$$R' = \frac{\sigma_f k(1-\nu)}{E\alpha}$$

Values for R' can be found in Table 1.12 and can be used for ranking without any additional calculations.

For smaller thermal shocks and preexisting flaws, resistance to crack propagation is expressed by

$$R'''' = \frac{E\gamma_s}{\sigma_f^2}(1-\nu)$$

Taking the surface energy to be 1000 for convenience, values of R'''' can be calculated and ranked in a similar fashion to R'.

The materials from Table 1.12 are ranked according the two criteria as follows:

Material	$R' = \frac{\sigma_f k(1-\nu)}{E\alpha}$ (kWm^{-1})	Material	$R'''' = \frac{E\gamma_s}{\sigma_f^2}(1-\nu)$ (arbitrary units)
Sintered WC (6% Co)	30	Hot-pressed BeO	6.60
Reaction-bonded SiC	18	Reaction-bonded Si_3N_4	2.79
Hot-pressed Si_3N_4	11	Reaction-bonded SiC	1.25
Reaction-bonded Si_3N_4	3.7	Hot-pressed Al_2O_3	1.17
Hot-pressed BeO	2.4	Hot-pressed Si_3N_4	0.31

Hot-pressed $A1_2O_3$	0.8	Sintered WC (6% Co)	0.23

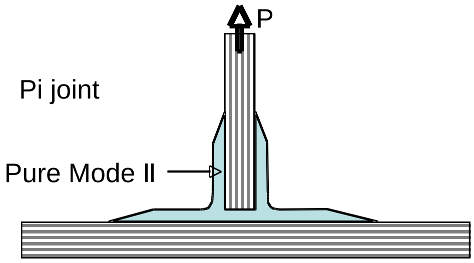
The material that ranks fairly high on both lists is Reaction-bonded SiC.

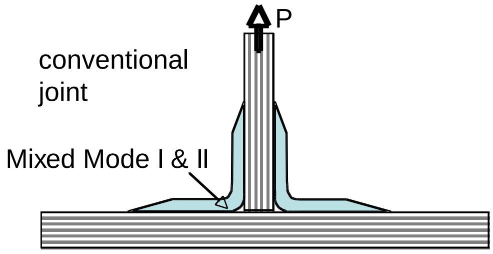
Extend

6.38 Search for an image of the Pi adhesive joint and sketch it. Label it in such a way that it is clear why the design is superior to simpler T-joints.

Answers will vary, but will probably include sketches and an explanation like this:

The Pi joint put the adhesive into nearly pure Mode II loading, which tends to be a high interface toughness condition. The conventional joint, however, has a region that is under mixed Mode I and Mode II loading with low load mixity (i.e., a high fraction of Mode I) which is a low toughness condition.





6.39 Search for a published paper (other than one referenced in this chapter) that uses the 4-point bend adhesion test configuration. Summarize the motivation for the work, and the major finding of the paper. Provide a full reference and a copy of the paper abstract.

Answers will vary.

6.40 Find a published paper that describes a product failure analysis and that uses one of the fracture mechanics principles introduced in this chapter. Summarize the failure scenario and the major finding of the paper. Report on the fracture mechanics equation from this chapter that is used in the failure analysis. Provide a full reference and a copy of the paper abstract.

Answers will vary.

CHAPTER 7

Review

7.1 Define *strength* and *toughness*. What is the typical trend between these two material properties when the microstructure of a given material is modified?

Strength is resistance to plastic deformation. Toughness is resistance to fracture. As microstructure is modified to increase strength, toughness typically decreases, and vice versa.

7.2 Under what circumstances might an engineer be willing to choose a low-strength material in exchange for high toughness? When might this not be the case?

When fracture under low-energy conditions (i.e., low stress and/or low strain) is not tolerable because of potential loss of human life or very high cost, toughness may be so important that a low strength material can be suitable. In these cases, it is generally possible to design with larger cross-sections to improve load-carrying capacity without unacceptable additional cost or loss of functionality. Structures like bridges and nuclear power plants are good examples. On the other hand, if very high strength is necessary and low mass is either highly desirable (as in a transportation application) or absolutely necessary for the function of the structure or device, some sacrifices in material toughness may be required. In these cases, it may be particularly important to promote fracture resistance through careful design (i.e., through redundancy and/or elimination of stress concentrations).

7.3 Without consulting any source other than a periodic table, what relative level of toughness would you expect for Ti vs. TiN? Justify your answer based on the probable types of atomic bonding found in these two materials.

Titanium metal is unquestionably held together by metallic bonds. Titanium nitride is a compound made from materials on the opposite ends of the periodic table. Their electronegativity values are therefore quite different, and the bonding that exists between them will have a fairly high degree of ionic character. Based on these differences in atomic bonding, one might expect the toughness of Ti to be greater (perhaps much greater) than that of TiN. In fact, this is the case: 45-75 MPa$\sqrt{m}$ is a reasonable range for Ti and its alloys, whereas 3-6 MPa$\sqrt{m}$ is a reasonable range for TiN. An order of magnitude difference!

7.4 Summarize the differences between *intrinsic toughening processes* and *extrinsic toughening processes* with regard to (i) the basis for toughening, (ii) the zone of activity with respect to the crack tip, (iii) the effect on crack initiation and extension, and (iv) the influence on R-curve behavior.

Intrinsic toughening (IT) is generally due to energy dissipation through processes like plastic deformation. IT mechanisms act ahead of the crack tip. Materials with high IT are resistant *to crack nucleation and crack propagation, and provide a consistent resistance to crack growth (no R-curve behavior).*

Extrinsic toughening (ET) is related to reductions in the crack driving force (the stress intensity) rather than energy dissipation. Many ET mechanisms act in the wake of the crack rather than at the crack tip. These mechanisms have the greatest effect on crack extension rather than initiation, and lead to R-curve behavior (crack-length dependent toughness).

7.5 What is *crack tip shielding*, what role does it play in toughening, and what is the general means by which this role is accomplished?

"Crack tip shielding" describes a class of extrinsic toughening mechanisms. These mechanisms lower the driving force for crack propagation by reducing the local stress intensity factor in the region of the crack tip. This may be accomplished by various means, including microcracking that increases the local compliance, compressive residual stresses that counter applied tensile stresses, and bridging of unbroken microstructural elements that limit crack opening displacements.

7.6 What are the two general intrinsic toughening approaches that work for all metals?

Improving alloy cleanliness and microstructural refinement (i.e., reducing grain size and second phase dimensions).

7.7 Why would you expect a steel refined by the Bessemer process (air blown over or through the melt) to exhibit inferior fracture properties to a steel refined in a Basic Oxygen Furnace (in which pure oxygen is blown through the melt)?

The BOF process has several advantages over the Bessemer process. From the point of view of fracture toughness, the use of pure oxygen avoids the introduction of undesirable gaseous impurities. Of particular importance, when air is used as in the Bessemer process, the high levels of nitrogen would introduce nitride particles and interstitial nitrogen that would reduce alloy toughness.

7.8 Name five elements that should generally be removed from steel to improve toughness.

Sulfur, phosphorous, hydrogen, nitrogen, and oxygen.

7.9 Explain why removing certain precipitate-forming elements from Al alloys is particularly beneficial for aircraft applications, but other precipitate-forming elements are critical and should not be removed. Please be specific about the elements in question, and be sure to explain why this is more relevant for aircraft than for many other possible applications.

Strength is improved by the presence of small, closely-spaced precipitates. Large, widely-spaced precipitates do not contribute to strength in any significant way, but they do serve as crack nucleation sites. Fe and Si are particularly bad for the toughness of Al alloys, so they should be minimized for critical aircraft applications. However, without any precipitation hardening Al is not very strong, so Cu, Mg, and Zn must be present to add strength.

Thankfully, these three elements tend to form small precipitates when the chemistry and the heat treatment process are well controlled, so there is less loss of toughness associated with their presence then for Fe and Si. This is a particular issue for aircraft because (i) failure is not acceptable, and (ii) increasing section size to compensate for low strength increases mass and hurts fuel efficiency.

7.10 Summarize the way in which fine-grain microstructures behave with regard to low temperature and room temperature strength and toughness. Why not make all metals as small-grained as possible?

At low temperatures, finer grain structures exhibit a decreased DBTT and an improvement in the lower shelf energy as compared to more coarse-grained material. At room temperature the smaller grain size leads to improved strength and toughness. At elevated temperatures, however, small grain size is associated with increased creep so the benefits of decreased grain size depend on the intended use temperature.

7.11 Define *mechanical fibering* and list at least three metal forming processes that cause it. Sketch the type of fibering associated with each process. Why might this be beneficial?

Mechanical fibering is the development of an aligned grain structure in the direction of mechanical working. It is caused by forming processes such as rolling, drawing, or swaging (a combination of drawing and twisting). Rolling causes grains that are something like long pancakes. Drawing creates grains shaped like wires. Swaging causes spiral grain patterns. The grain boundaries may act as barriers to crack growth, so proper alignment with respect to the crack growth direction can cause an increase in toughness compared to the unaligned material.

7.12 Is the laminated glass that is used for automobile windshields best described in terms of the *arrester*, *divider*, or *short transverse* orientation?

The arrester orientation. Cracks typically begin in the outer layer of glass, propagate to the inner polymer layer, and arrest. The glass/polymer interface is perpendicular to the typical crack growth direction.

7.13 Fig. 7.31 depicts 6 possible Compact Tension specimens cut from the same rolled plate. Identify the pairs of CT specimen orientations that should exhibit nearly the same fracture toughness, and then identify which pair of orientations should be toughest and the least tough of the set.

The grains in the rolled plate can be pictured as elongated pancakes with their faces parallel to the top and bottom surfaces of the plate. The key to determining the equivalent pairs is to imagine a crack growing from the notch in each CT specimen. If the crack hits a periodic series of grain boundaries (i.e., the tops and bottoms of the "pancakes") then it is in the arrester orientation. If it would continuously cut grain boundaries (the sides of the "pancakes") it is in the divider orientation, and if it is likely to pass directly along grain boundaries (between the "pancakes") it is in the short transverse orientation. The similar pairs, then, are T-S:L-S (arrester, toughest, both with a crack perpendicular to the plate's top), L-T:T-L (divider), and S-T:S-L (short-transverse, least tough, both with a crack running along the grain boundaries). There are small differences in toughness within the pairs *because the grain structure is flattened and elongated, so the L and T directions (for instance) are not truly identical.*

7.14 What class of toughening mechanisms is usually at work in engineered ceramics, and why is this the case?

Extrinsic toughening mechanisms, because ceramics at room temperature have few options for improving intrinsic toughness.

7.15 Identify one thing that *thermally tempered* and *chemically tempered* glass have in common, and one critical difference.

There are many possible answers to this question. One likely common element is that both strengthening approaches depend on compressive residual stresses at the surface of the glass. Another similarity is that both raise the strength but do not prevent post-breakage instability (i.e., falling into many pieces). Differences include the method of manufacture, the nature of the residual stress profile, and the fact that thermally tempered glass cannot be cut or drilled successfully after tempering whereas chemically tempered glass can survive a cutting or drilling operation post-temper.

7.16 Identify which is typically tougher: a semi-crystalline or an amorphous polymer.

Semi-crystalline polymers are generally tougher due to the folded chain conformation.

7.17 Describe two general approaches to toughening an amorphous polymer. Why do they work?

The addition of a plasticizer in sufficient concentration will increase chain mobility and therefore increase the activity of energy dissipation mechanisms. Another approach is to add fillers such as rubber or glass particles. These may also introduce energy dissipation mechanisms such as debonding/cavitation at the polymer/particle interface, or may cause a reduction in driving force through crack deflection. Rubber particles or short glass rods can also serve as bridging elements in the crack wake.

7.18 Which condition encourages the fiber pull-process in a fiber reinforced composite: high fiber/matrix interface strength or low fiber/matrix interface strength?

Low interface strength is necessary for the fiber pull-out process to occur. If the interface is too strong, the crack will be forced to cut through the fiber and no bridging can happen.

7.19 Certain mollusk shells can be much tougher than others, but are not extremely tough compared to many engineered materials. Why, then, are there attempts to mimic the toughening mechanisms at work in natural materials such as these?

Tough mollusk shells are made from fairly poor materials but manage to achieve reasonable toughness thanks to the presence of certain microstructural features. If the same features can be duplicated in engineered structures, but with better starting materials, very high toughness *can be expected.*

7.20 Explain the structural origin behind the anisotropic toughness of cortical bone, and identify the toughest and least tough crack orientations for a long bone.

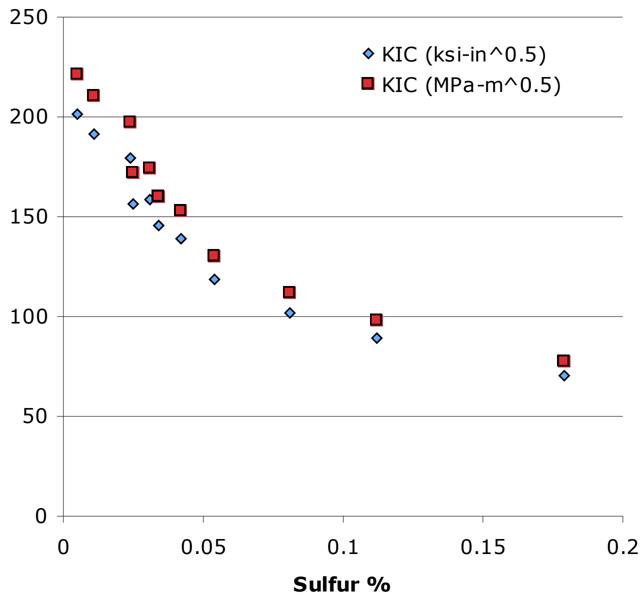
Cortical bone is built from approximately-cylindrical bundles called osteons. These osteons are joined along "cement lines" that are preferentially aligned with a certain axis of a bone. In a long bone such as the femur or tibia, the osteons are aligned along the long axis. Cracks beginning at the surface of a bone in bending are typically in an arrester-type orientation, stopping at cement lines, so there is maximum toughness for this scenario. If a crack can be formed on the surface aligned with the long axis and propagated inwards, however, it will pass along the cement lines. This would be the least tough orientation. Thankfully, this is an unusual cracking mode for a bone.

Practice

7.21 The severe effect of sulfide inclusions on toughness in steel is shown in Fig. 7.7, where the Charpy V-notch (CVN) upper shelf energy drops appreciably as sulfur content increases. Compute the fracture toughness level K_{Ic} at approximately 25 °C (77 °F) for these 11 alloys using the Barsom-Rolfe relation described in Chapter 6. Plot the fracture toughness as a function of sulfur fraction.

The following data was extracted from the figure, so the numbers may not be exact. The trend, however, should be correct.

Sulfur %	CVN ft-lb	KIC ksi-in^0.5	KIC MPa-m^0.5
0.005	77	201	221
0.011	72	191	210
0.025	55	156	172
0.024	66	179	197
0.031	56	159	174
0.034	50	146	160
0.042	47	139	153
0.054	38	119	130
0.081	31	102	112
0.112	26	89	98
0.179	19	70	77



7.22 For a stress level of 240 MPa compute the maximum stable radius of a semicircular surface flaw in 7075-T651 aluminum alloy plate when loaded in the L-T, T-L, and S-L orientations. Assume plane-strain conditions.

	K_{IC}	σ_{ys}
L-T	*29*	*537*
T-L	*26.5*	*520*
S-L	*18.5*	*472*

$$K_{IC} = 1.1\left(\frac{2}{\pi}\right)\sigma\sqrt{\pi a}$$

$$29 = 1.1\left(\frac{2}{\pi}\right)240\sqrt{\pi a};\, a_{L-T} \approx 9.5\ mm$$

$$26.5 = 1.1\left(\frac{2}{\pi}\right)240\sqrt{\pi a};\, a_{T-L} \approx 7.9\ mm$$

$$18.5 = 1.1\left(\frac{2}{\pi}\right)240\sqrt{\pi a};\, a_{S-L} \approx 3.9\ mm$$

7.23 You are given a 10-cm-diameter cylindrically shaped extruded bar (length = 50 cm) of an aluminum alloy.

(s) What specimen configurations would you use to characterize the degree of anisotropy of the material's fracture toughness?

The extruded bar should exhibit anisotropic fracture toughness behavior with the lowest K_{IC} value associated with the growth of a radial crack – corresponding to a disk-shaped compact specimen [DC(T)] (see Type 2, Appendix B); the highest K_{IC} values would be associated with an axially loaded sample with a crack oriented perpendicular to the stress axis [similar to the L-T orientation]. The latter *test could be conducted with a bend bar (Type 6) or single edge-notched specimen (Type 7) (see Appendix B).*

(t) If the lowest fracture toughness of the bar was found to be 20 $\text{MPa}\sqrt{\text{m}}$, what would be the load level needed to achieve this stress intensity level?

The minimum fracture toughness could be determined with the DC(T) sample with

$$K = \frac{P}{BW^{0.5}} \cdot f(a/w)$$

$$f(a/W) = \frac{(2 + a/W)}{(1 - a/W)^{3/2}}[0.76 + 4.8(a/W) - 11.58(a/W)^2 + 11.43(a/W)^3 - 4.08]$$

Given that K_{IC} = 20 MPa$\sqrt{m}$, we find that P = 5676 N

(u) Confirm the presence of plane strain conditions, given that σ_{ys} = 500MPa. (Assume specimen thickness = 1 cm, *a/W* = 0.5, and bar diameter = 1.2 W.)

For plane strain conditions,

$$t\&a \geq 2.5\left(\frac{K_{IC}}{\sigma_{ys}}\right)^2$$

$$2.5\left(\frac{20}{500}\right)^2 = 0.004\ m$$

Plane strain conditions exist.

7.24 A particular pressure vessel is fabricated by bending a rolled aluminum alloy plate into a cylinder then welding on end caps. The alloy used for the cylinder has a distinct layered structure from the rolling process. The rolling direction is around the circumference of the cylinder. The plate thickness is 3 mm. The measured fracture toughness values for the different orientations of this material are shown in Example 7.1 and figure 7.31. The internal pressure leads to a hoop stress of 300 MPa.

(v) If two identical semicircular cracks are initiated on the inner surface of the cylinder such that one is growing along the cylinder length and the other across the cylinder width, which would be more likely to lead to fast fracture?

The L-T orientation is slightly tougher than the T-L. However, the L-T crack lies along the cylinder axis and is therefore opened by the hoop stress. This stress is twice the longitudinal stress acting on the T-L orientation, and fracture toughness is directly proportional to stress. Thus the L-T crack is favored to fail first.

(w) If a circular embedded penny crack was created internally during fabrication so that the crack lies in the S-T orientation, will this fail before either of the cracks in part (a)?

This crack would be opened by a radial tension stress. There is no radial stress in the radial direction of a pressure vessel, and the strain is compressive, so there is no chance that this crack will fail despite being in the least tough orientation.

(x) If only the longitudinal crack was present, would this design meet a leak-before-break criterion?

One possible approach is to find the critical crack length and compare it to the wall thickness. The appropriate fracture toughness in this case is the L-T value.

$$K_{IC} = Y\sigma\sqrt{\pi a}$$

$$a_c = \frac{1}{\pi}\left(\frac{K_{IC}}{Y\sigma}\right)^2 = \frac{1}{\pi}\left(\frac{29.7\ MPa\sqrt{m}}{(1.12)(2/\pi)(300\ MPa)}\right)^2 = 6.14\times10^{-3}\ m = 6.14\ mm$$

The plate is 3 mm thick, so it easily meets the leak-before-break condition.

7.25 For lack of a suitable material supply, a thin-walled cylinder is machined from a thick plate of 7178-T651 aluminum alloy such that the cylinder axis is oriented parallel to the rolling direction of the plate. If the cylinder's diameter and wall thickness are 5 cm and 0.5 cm, respectively, determine whether the cylinder could withstand a pressure of 50 MPa in the presence of a 0.2-cm-deep semicircular surface flaw.

For this specimen orientation, the hoop stress is applied normal to the S-L direction. We see from Table 8.8 that K_{IC} = 17 MPa$\sqrt{m}$. The hoop stress is given by

$$\sigma = \frac{Pr}{t} = \frac{50(2.5)}{0.5} = 250\ MPa$$

The stress intensity factor is

$$K = 1.1\left(\frac{2}{\pi}\right)\sigma\sqrt{\pi a}\times\sqrt{\sec\frac{\pi a}{2t}}$$

$$K = 1.1\left(\frac{2}{\pi}\right)(250)\sqrt{\pi(0.002)}\times\sqrt{\sec\frac{\pi(0.002)}{2(0.005)}} = 15.4\ MPa\sqrt{m}$$

The cylinder would barely withstand the applied pressure what with the prevailing stress intensity level being 91% of K_{IC}. For continued operation of this cylinder, either the defect would have to be repaired or the applied pressure reduced substantially.

7.26 Ordinary sheets of borosilicate glass are tested in bending, and are found to fracture at an average stress of 72 MPa. After thermal tempering, the stress at failure increases by 90%. What is the sign and magnitude of the stress induced in the glass by the tempering operation?

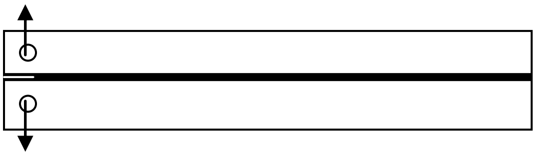
Tempering induces compressive stresses near the glass surface that can counteract the applied tensile stresses that cause failure. The effective Crack Opening Stress (COS) is

$$\sigma_{COS} = \sigma_{appl} + \sigma_r + \sigma_{c/p}$$

There is no mechanical prestress on the glass, so the third term ($\sigma_{c/p}$) is irrelevant. The fundamental behavior of the glass is not changed by the tempering process, so the COS at *fracture does not change. The residual stress induced by the tempering process must therefore be*

$\sigma_r = \sigma_{COS} - \sigma_{appl} = 72\ MPa - 1.9(72\ MPa) = -64.8\ MPa$

7.27 The quality of the bond between an epoxy and a metal is sometimes measured using a Double Cantilever Beam (DCB) test, as depicted below. In this case, two metal bars are epoxied together, a pre-crack is created, then the DCB specimen is gradually pulled apart using pins inserted through holes drilled in the metal bars. Assume that the precrack is 10 mm long and the metal bars are each 150 mm long x 10 mm wide x 5 mm thick. Also assume that the fracture is cohesive. Imagine that in this case the epoxy has typical characteristics for this class of materials (T_g > 20 °C, E ~ 2.4 GPa, ν ~ 0.3, σ_{TS} ~ 27 MPa), and that the bars are hardened steel.



(a) If the fracture toughness is determined to be 0.8 MPa√m, what is the strain energy release rate for this joint? Be sure that the units for your answer include an energy term.

Chapter 6 has the necessary conversion equation:

$$G_C = \frac{K_C^2}{E} = \frac{\left(0.8\ MPa\sqrt{m}\right)^2}{2400\ MPa} = 2.67 \times 10^{-4}\ MPa \cdot m$$

$$G_C = 267\frac{N}{m^2} \cdot m = 0.267\frac{kJ}{m^2}$$

(b) If the temperature is raised high enough that some degree of plasticity becomes possible in the epoxy at the strain rate used in the test, would the fracture toughness probably *increase*, *decrease*, or *remain unchanged*? Explain, and provide a simple equation to support your answer.

The Orowan and Irwin extension of Griffith's relation says that the fracture toughness is related to energy dissipated in surface formation and in plastic deformation, so increased plasticity would be expected to raise the toughness.

$$K_C = \sqrt{2\left(\gamma_S + \gamma_P\right)}$$

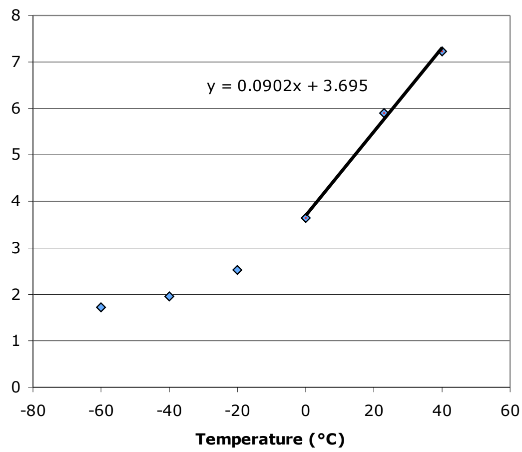
(c) Would you be better off under the circumstances adding an adhesion promoter to the epoxy/metal interface or adding rubber particles to the epoxy? Why?

The fracture is described as cohesive, which means the crack runs through the epoxy, not at the interface. There would be no benefit to improving the strength of the interface because it is not the weakest link; adding rubber particles to the epoxy would be much more beneficial.

(d) If you were to add rubber particles, and the behavior of the reinforced epoxy is similar to that summarized in Table 7.7, what is the critical strain energy release rate expected for the DCB specimen tested at 30 °C? For the scenario described above, what would be the failure load in this case?

First, plot critical strain energy release rate vs. temperature to get a trend, then interpolate to get an estimate for the energy release rate at 30 °C. It looks fairly linear over the temperature range of interest, so it is possible to fit a straight line to the last three data points. Then

$$G_{IC} = 0.0902T + 3.695 = 0.0902\left(30\,°C\right) + 3.695 = 6.401\,kJ/m^2$$



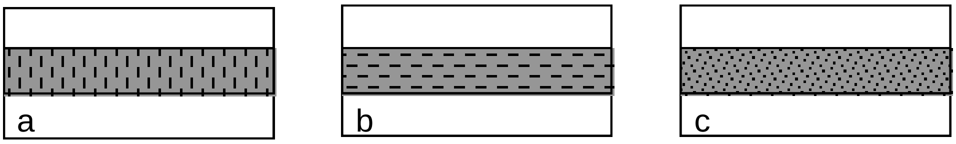
With this value, it is possible to use Eq. 6-87 to solve for the critical load:

$$G_{IC} = \frac{12P^2a^2}{b^2h^3E_s}$$

$$P = \sqrt{\frac{G_{IC}b^2h^3E_s}{12a^2}} = \sqrt{\frac{\left(6.4\times10^3\,\frac{N}{m^2}\cdot m\right)\left(10\times10^{-3}\,m\right)^2\left(5\times10^{-3}\,m\right)^3\left(2.4\times10^9\,\frac{N}{m^2}\right)}{12\left(10\times10^{-3}\,m\right)^2}} = 400\,N$$

(e) A clever engineer has an idea to improve the fracture toughness of this joint by coating the metal with a layer of carbon nanotubes (CNTs) before flowing the epoxy. What would be the most advantageous orientation of the CNTs with respect to the joint if you assume they are rigid cylinders? Explain.

The crack will run from left to right, so the most advantageous CNT orientation would be the one depicted in (a). The CNTs could act as barriers to crack propagation, and might also contribute to energy dissipation through "fiber" pull-out. The other two orientations add little in the way of barriers and may even make things worse if the interface between the epoxy and the CNTs is not strong.



(f) How would you best engineer the interface between the CNTs and the epoxy for the greatest toughness: *excellent* adhesion, *terrible* adhesion, or *moderate* adhesion? Explain.

Moderate adhesion would be the best choice. Excellent adhesion would pass up opportunities for dissipation mechanisms associated with debonding. Terrible adhesion might promote debonding, but the energy involved would be so low as to be negligible. Moderate adhesion would allow the CNTs to reinforce the epoxy at low loads, then to debond and pull out at higher loads.

Design

7.28 A rectangular component is to be fabricated from the least expensive steel available (presumed to be the alloy with the least alloying additions). The final decision is to be made between 4330V (425°C temper) and 9-4-20 (550°C temper). Which alloy would you choose if the component is to experience a stress of half the yield strength in the presence of a quarter-circular corner crack with a radius of 10 mm? Would you answer be the same if the design stress were increased to 65% of the yield strength? The properties of these materials are given in Table 8.8.

4330V (425ºC)		*9-4-20 (550ºC)*	
σ_{ys}	K_{IC}	σ_{ys}	K_{IC}
1315	*103-110*	*1280-1310 (1295 avg))*	*132-154*

$$\sigma_{design} = \frac{\sigma_{ys}}{2} \text{ or } 0.65\sigma_{ys}$$

$$a = 10\ mm$$

$$K = 1.12^2\left(\frac{2}{\pi}\right)\sigma_d\sqrt{\pi a}$$

For 4330V

$$K = 1.12^2\left(\frac{2}{\pi}\right)\left(\frac{1315}{2}\right)\sqrt{\pi(10^{-2})} = 93\ MPa\sqrt{m}$$

For 9-4-20

$$K = 1.12^2\left(\frac{2}{\pi}\right)\left(\frac{1295}{2}\right)\sqrt{\pi(10^{-2})} = 91.6\ MPa\sqrt{m}$$

Both materials show K levels < K_{IC}; therefore, neither one of them will fracture.
Choose 4330V since it is cheaper (though margin of safety is smaller)

For $\sigma_d = 0.65\ \sigma_{ys}$
4330V has K = 121 MPa$\sqrt{m}$ which is higher than K_{IC}
9-4-20 has K = 119 MPa$\sqrt{m}$ which is lower than K_{IC}
Therefore, choose 9-4-20 steel

7.29 A design for a pair of adjacent buildings calls for a ground-level, enclosed connecting walkway with large plates of glass for the walls and the ceiling. Considering the conditions associated with heavy use of the walkway, the potential for falling objects from above, and safety of the users, would you suggest using laminated or tempered glass for the walls? For the ceiling? If the architect calls for tempered glass for one of the locations, but wants a complicated saw-tooth prism cut deep into the glass before or after it is tempered, what is the right order for processing (tempering first vs. cutting first) and which is more likely to succeed: thermal tempering or chemical tempering?

Although there are several ways to answer this question, one reasonable response is to choose tempered glass for the walls and laminated glass for the ceiling. The walls are likely to experience daily abuse from people and objects brushing against them, and objects from outside are likely to hit the walls, so very high scratch and fracture resistance is critical. If a panel does break, the small pieces will fall to the ground and the likelihood of injury is very small. The ceiling is less likely to need terrific fracture resistance, but in the event of a blow from a falling object safe-breakage behavior is important; it is critical to avoid dropping shards of glass on the people below. Laminated glass is probably best for this location since it will retain some integrity even after fracture.

Cutting a prism shape into glass after tempering is a bad idea for two reasons. In the case of thermal tempering, the cutting will cause the glass to explode. In the case of chemical tempering, the cutting will remove the tempered layer just where you need it the most — where the prism will create lines of stress concentration. Cutting must therefore occur before tempering, regardless of which technique is used. Of the two, chemical tempering is the better choice for shapes with varying thickness. The cooling rate is critical for successful thermal tempering, and it would be very difficult to create a consistent cooling rate in a glass that varies significantly in thickness.

Extend

7.30 Find an example of a product (or a proposed product) made from Partially Stablized Zirconia. Explain why the properties associated with PSZ are important for your chosen application. Provide a clear reference to the source of your information.

Answers will vary widely.

7.31 Find an image of a quench pattern (a.k.a. quench marks) in a tempered glass plate (or, if ambitious, make such an image using a camera and an appropriate lens). Provide a full reference for the image.

Answers will vary widely.

7.32 Write a short report identifying the closest nuclear power plant to your hometown. Include information on the license, when it was last renewed, and its end-date.

Answers will vary widely.

CHAPTER 8

Review

8.1 In the phrase "subcritical flaw growth mechanisms", what does *subcritical* mean?

Failures occur when the stress intensity is at the point where $K = K_{IC}$, i.e., where the applied stress intensity is equal to the critical stress intensity factor (the fracture toughness). If the combination of applied stress and crack length is such that $K < K_{IC}$, then the crack is subcritical.

8.2 What do the acronyms EAC, SCC, HE, and LME stand for?

Environment-assisted cracking (EAC), stress corrosion cracking (SCC), hydrogen embrittlement (HE), and liquid-metal embrittlement (LME)

8.3 What is the main difference between the HEAC model and the IHAC model?

The source of the hydrogen is the main difference. In the hydrogen-environment-assisted-cracking (HEAC) model, hydrogen is not initially present at the crack tip. It is introduced there (or nearby) as a result of processes such as hydrogen-generating corrosion reactions taking place at the crack tip. In the internal-hydrogen-assisted-cracking (IHAC) model, dissolved hydrogen is already present in the material.

8.4 What EAC phenomenon is sometimes associated with welding?

Time-delayed cold cracking due to hydrogen embrittlement can occur in welds that have picked up hydrogen while in the hot state.

8.5 What is the objective of a post-weld heat treatment with regard to EAC?

Post weld heat treatments can reduce the residual stresses associated with metal shrinkage during the welding process, and can therefore reduce the tendency for EAC.

8.6 What three major factors affect the hydrogen-embrittling process?

The original location and form of hydrogen, the transport reactions involved in moving the hydrogen to locations in the metal where it causes embrittlement, and the nature of the embrittling mechanism.

8.7 What three simultaneous conditions must be satisfied for SCC?

For SCC to occur, one needs sufficient and sustained tensile stresses, a susceptible material, and aggressive environment relative to the material.

8.8 SCC is often described in terms of a film rupture model. What is the film the model refers to? Provide a specific example in support of your answer.

The film in question is a reaction product, such as an oxide, that naturally develops on the surface of many metals in certain environments. Examples include Al_2O_3 for Al alloys, Cu_2O for Cu alloys, and Cr_2O_3 for stainless steel alloys.

8.9 How is it that two solid metals in contact at a temperature below the melting point of either can lead to liquid metal embrittlement?

Thermodynamically speaking, there is always an equilibrium vapor pressure for any solid or liquid. That is, a small amount of solid or liquid is always evaporating (and probably recondensing as well). For certain metal pairs, the vapor phase of the embrittling metal migrates by surface diffusion to the crack tip in the embrittled solid where it participates in the LME cracking process.

8.10 What controls the rate of Dynamic Embrittlement, and what mathematical dependence on temperature does this suggest?

Dynamic embrittlement (DE) is controlled by the diffusion of embrittling elements to grain boundaries. This suggests that the rate of DE cracking should be proportional to the diffusivity of the embrittling element(s), so that

$$DE\ rate \propto e^{\frac{-\Delta H}{RT}}$$

8.11 What are the similarities and differences between K_{ISCC} , K_{IEAC} and K_{EAC}? How do they differ from K_{IC}?

All three are K threshold values for the onset of EAC processes and subcritical flaw growth (rather than a threshold for the onset of unstable mechanical cracking, which is the meaning of K_{IC}). Both K_I values are determined under plane strain loading. K_{ISCC} is for Stress Corrosion Cracking specifically, whereas K_{IEAC} is for EAC in general, indicating a philosophical point of view that EAC is a multi-phenomenon failure mode. K_{ISCC} is also a threshold value, but determined in plane stress.

8.12 When EAC processes are active, under what condition does final fracture occur?

Failure occurs when K approaches K_{IC}. The fundamental fracture toughness of the material is not affected by the environment; instead, small cracks grow under sustained loads to the point where the environment-insensitive critical stress-intensity-factor level is approached.

8.13 Large transport aircraft make wide use of Al alloys with a -T6 temper (which indicates the peak strength condition). However, for certain locations in these aircraft it is preferred to use Al alloys with a -T7 temper (which indicates an overaged condition). What could possibly motivate this design decision even though it probably means an undesirable increase in vehicle weight?

The –T7 overaged temper condition has been found to improve the EAC resistance of many Al alloys. For certain aircraft components the resistance to EAC may be more critical than the degree of weight savings associated with a stronger alloy.

8.14 EAC in metals causes cracks to form and grow. What other feature can form and grow in amorphous polymers suffering from EAC?

Crazes tend to initiate and grow at lower stresses than usual under EAC conditions.

8.15 What two changes occur with regard to EAC behavior of PVC and polyethylene water and gas transmission pipelines as temperature is increased?

As the temperature increases, the lifetime for a given stress level decreases. Also, at a given stress level a rise in temperature may cause a shift in failure mode from one that is largely elastic until failure (See Fig. 8.23, Region B) to one that involves extensive plastic deformation (See Fig. 8.23, Region A).

8.16 What fracture surface feature is often a clue for the presence of environment-assisted cracking in polyethylene water pipe systems?

Environment-assisted cracking in polyethylene often results in the development of a tufted fracture surface appearance (Fig. 8.24).

8.17 Polycarbonate is sometimes used for protective helmets, such as motorcycle helmets. However, it is often not advisable to apply adhesive stickers to the helmet surface or to clean the surface with a solvent, as both can potentially degrade the helmet strength. Why might this be?

Adhesive stickers and cleaning solvents both expose the PC material to chemicals that can cause EAC. Stresses resulting from a crash, or even the residual stresses from the helmet manufacturing process, can cause premature failure after exposure to certain chemicals.

8.18 Ceramic and glass materials are routinely used to contain chemicals, including some that are quite aggressive. Are these materials therefore immune to EAC?

Unfortunately, ceramics and glasses can be susceptible to EAC just like metals and polymers.

Practice

8.19 A wrought, high strength steel known to be susceptible to hydrogen embrittlement is used to fabricate a component. The component is machined from thick plate with no regard as to the component orientation, relative to the plate's rolling plane and direction.

It is determined that only a subset of components are failing in the field due to hydrogen embrittlement. How could this be?

Inclusions in plate products are typically aligned along the rolling plane and in the rolling direction. As discussed, hydrogen tends to accumulate at grain boundaries and inclusion-matrix interfaces. As such, inclusion stringers and elongated grain boundaries in the rolling direction provide attractive crack paths if the loading is perpendicular to these microstructural features.

8.20 The same alloy is used to fabricate two different components, both of which result in identical mechanical properties. Cold working yields a microstructure with a much higher incidence of symmetrical grain boundaries for Component 1 as compared with that of Component 2. Which item will most likely have a higher resistance to dynamic embrittlement?

Component 1 contains a much higher incidence of symmetrical grain boundaries than Component 2 and will, therefore, be more resistant to dynamic embrittlement.

8.21 An investigation was made of the rate of crack growth in a 7079-T651 aluminum plate exposed to an aggressive environment under a static stress σ. A large test sample was used with a single-edge notch placed in the T-L orientation. As indicated in the accompanying table, the rate of crack growth under sustained loading was found to vary with the magnitude of the applied stress and the existing crack length. The material exhibits Regions I and II EAC but not Region III. If the K_{IC} for the materials is 20 $\text{MPa}\sqrt{\text{m}}$ how long would it take to break a sample containing an edge crack 5 mm long under a load of 50 MPa? *Hint:* First establish the crack growth rate relations.

Cracking Rate (m/sec)	**Applied Stress (MPa)**	**Crack Length (mm)**
10^{-9}	35	5
32×10^{-9}	35	10
1×10^{-6}	70	5
1×10^{-6}	70	7.5

da/dt rises rapidly in stage 1 to 10^{-6} m/s, corresponding to K level of 9.65 $MPa\sqrt{m}$, i.e.

$$K = 1.1\sigma\sqrt{\pi a} = 1.1(70)\sqrt{\pi(0.005)}$$

When σ = 50 MPa and a = 0.005 m, the initial K level = 6.89 $MPa\sqrt{m}$ which will involve stage I growth and will persist until crack length reaches 0.0098 m, i.e.

$$9.65 = 1.1(50)\sqrt{\pi a}; a = 0.0098\ m$$

From analysis of the data, Stage I growth may be described by $da/dt = CK^m$ where m is found to be about 10 and $c \approx 1.54 \times 10^{-16}$. The time spent in Stage I growth is found by integration of

$da/dt = 1.54 \times 10^{-16} K^{10};\ where\ K = 1.1\sigma\sqrt{\pi a}$

$$t_{stage\ I} = \frac{1}{4(1.54 \times 10^{-16})(1.1\sqrt{\pi})^{10}(50)^{10}}\left[\frac{1}{(0.005)^4} - \frac{1}{(0.0098)^4}\right] = 3.12 \times 10^4\ seconds$$

For $a > 0.0098\ m$, da/dt *is constant and equals* $1 \times 10^{-6}\ m/sec$ *with Stage II persisting to final fracture*

$K = 1.05\sigma\sqrt{\pi a}$

$20 = 1.05(50)\sqrt{\pi a}$

$a = 0.0462;\ Note\!:smaller\ surface\ flaw\ correction$

$$t_{stage\ II} = \frac{\Delta a}{10^{-6}} = \frac{0.0462 - 0.0098}{10^{-6}} = 3.64 \times 10^4\ seconds$$

$t_{total} = 6.76 \times 10^4\ seconds = 18.8\ hours$

8.22 For the 18 Ni (300)-maraging steel listed in Table 8.3, calculate the stress level to cause failure in a center-notched sample containing a crack 5 mm long. What stress level limit would there have to be to ensure that EAC did not occur in a 3.5% NaCl solution?

For this alloy, $K_{IC} = 80\ MPa\sqrt{m}$ *and* $K_{IEAC} = 8\ MPa\sqrt{m}$. *Fracture will occur when*

$K = \sigma\sqrt{\pi a}$

$80 = \sigma\sqrt{\pi(0.0025)}$

$\sigma = 903\ MPa$

To avoid EAC, the stress should be kept below 90.3 MPa, i.e.

$8 = \sigma\sqrt{\pi(0.0025)};\ \sigma = 90.3\ MPa$

8.23 How much faster than the room temperature value would a crack grow in a high-strength steel submerged in water if the temperature were raised 100°C?

From Section 8.2.1.3, the activation energy is approximately 38 kJ/mole. Since

$da/dt \propto e^{\Delta K/RT}$

Change in growth rate is

$$\frac{e^{-\frac{38000}{8.33(300)}}}{e^{-\frac{38000}{8.33(400)}}} = 44.8 - fold\ increase$$

8.24 A metal plate is found to contain a single-edge notch and is exposed to a static stress in the presence of an aggressive environment. Representative data obtained from crack-growth measurements are given in the following table:

Measurement	Cracking Rate (m/s)	Applied Stress (MPa)	Crack Length (mm)
1	1×10^{-9}	30	5
2	4.1×10^{-9}	30	8
3	8×10^{-9}	30	10
4	6.4×10^{-8}	60	5
5	6.4×10^{-8}	60	6
6	6.4×10^{-8}	60	7

(a) What is the growth rate relation among the cracking rate, stress, and crack size?
(b) Does the relation change? If so, why?
(c) What was controlling the cracking process in the regime associated with measurements 4, 5, and 6?

For this configuration, $K = 1.1\sigma\sqrt{\pi a}$*. For the 6 data points, K is computed to be 4.14, 5.23, 5.85, 8.27, 9.06, and 9.79. For data points 1-4, K doubles and* da/dt *increases by 64-fold. This suggests a relationship of the form:* $da/dt = CK^6$*. Using the data,* $C = 1.99 \times 10^{-13}$*. This represents Stage I behavior. Stage II corresponds to a constant growth rate of* 6.4×10^{-8} *m/sec (controlled by temperature).*

Design

8.25 As-welded, austenitic stainless steel connections are experiencing repeated failures in beachfront properties in Miami, Florida due to chloride-induced stress corrosion cracking. No such failures have been reported to date in Chicago, Illinois, the other city where these connections were installed. Write a brief memo to your supervisor with an explanation of the regional dependence of this phenomenon, and propose two potential approaches to mitigate the risk for this type of cracking by changing both fabrication methods and material of construction.

Miami is typically hot and humid, and represents a chloride-containing environment due to its proximity to the salt water of the Atlantic Ocean. Chicago is situated on Lake Michigan and represents a cooler climate on a fresh water lake, which explains the lack of chloride-induced SCC in this environment. Consider changing the material of construction to an alloy that exhibits a higher resistance to SCC in the presence of chlorides, possibly a high-nickel alloy. Consider a post weld heat treatment to reduce residual stresses resulting from the welding process that will also reduce SCC susceptibility.

8.26 You are a design engineer for a housewares company that is looking to produce glass flower vases. You are told that the factory that your company usually contracts to do this type of work has had intermittent problems in the past controlling their oven temperature. In addition, the manufacturer has asked you to choose between soda-lime and silica glass for your product. What are your concerns, and what decisions should you make in response?

If oven temperatures are not controlled during the annealing process, residual stresses can be introduced that can increase the likelihood of environmental assisted cracking. Therefore, first audit the facility and make sure that the company has implemented new measures that will catch any process deviations, including oven temperature variations. If they have not, find another vendor who can accurately control their manufacturing process. Between the two materials available, silica glass will have the lower crack velocity at room temperature in aqueous environments for a given stress intensity factor and would be better for this application on that basis.

Extend

8.27 Find a practical example of stress corrosion cracking failure of a component (or a class of components) made from a brass alloy. Summarize the failure circumstances and the failure analysis. Include copies of photographs, if they are available. Provide a full reference.

Answers will vary widely.

CHAPTER 9

Review

9.1 What are clamshell/beach markings? Explain their significance in terms of the conditions experienced during the life of a failed component, and their value in failure analysis.

Clamshell/beach markings are curved lines that often appear on a fatigue fracture surface. They indicate changes in load or environment conditions that occurred during the lifetime of the component. Since these markings often are curved, with their centers of curvature at the fatigue crack origin, if they are present they serve as a useful guide to direct the investigator to the fracture initiation site. It is important to note that a fatigue fracture surface need not reveal clam shell markings when load levels and the environment remain unchanged.

9.2 What are *ratchet lines*, and how are they arranged with respect to the fatigue crack front and any clamshell marks that may also be present?

Ratchet lines are actually steps in fatigue fracture surface that stem from the interaction between multiple fatigue crack initiation sites. As the sites are unlikely to be on exactly the same plane of the component, the cracks emanating from them must form steps when then merge to form one large crack. The ratchet lines therefore separate one planar fracture region from its neighbor. The lines are perpendicular to the fatigue crack front and any clamshell marks that may also be present.

9.3 Why is it safe to assume that the railroad wheel axles studied by Wöhler are better described by stress-controlled cyclic loading than by strain-controlled cyclic loading?

The magnitude of the load is largely determined by the mass of the railroad car and its contents. The cyclic nature of the load comes from rotation of the axles so that a particular location alternates between tension and compression. There is nothing that mechanically limits the strain to a certain value, so stress-controlled cyclic loading is closer to the real situation than strain-controlled cyclic loading.

9.4 What role can a notch or a flaw play in determining the total fatigue life of a component?

Fatigue life can be reduced drastically by the presence of a notch. The total fatigue life of a component may be viewed as a three-stage process involving initiation, propagation, and final failure stages. The total number of cycles can be described as

$$N_{total} = N_i + N_p$$

When design defects or metallurgical flaws are preexistent, the initiation stage (N_i) *is* *shortened drastically or completely eliminated, resulting in a reduction in potential fatigue life.*

9.5 What does it mean to conduct a fatigue test with R = -0.5?

R ≡ minimum load/maximum load (or minimum stress/maximum stress), so $P_{min}/P_{max}=-0.5$*, or* $P_{min}=-0.5P_{max}$*. This means that* P_{max} *is in tension and* P_{min} *is in compression, and that the magnitude of* P_{max} *is twice as large as that of* P_{min}*.*

9.6 What are the *S* and N axes on an *S-N* diagram, and what does the region to the left and below the plotted line indicate?

S is the stress change, usually the stress amplitude (half the total stress range), and N is the number of complete load cycles. The region to the left and below the plotted line indicates combinations of cyclic stress and number of cycles for which no fatigue failure is expected to occur. Note that this does not mean that there is no fatigue damage — a crack may be growing, but it has not reached the critical length for unstable propagation.

9.7 What is a *fatigue limit*, and what relevance does it have for lifetimes in the gigacycle (10^9) regime? Explain.

A fatigue limit is an apparent alternating stress level below which no fatigue failure will ever occur. This may be true if only one fatigue damage mechanism is possible, but at very high cycles additional mechanisms may come into play so that failure can still occur even for stresses below the "fatigue limit".

9.8 What is the typical effect of increasing mean stress on fatigue life?

Increasing mean stress (even for a given maximum stress) typically causes a reduction in fatigue life.

9.9 Automobile manufacturers specify a certain number of miles at which the engine timing belt should be replaced. Is this an example of a "safe-life"-designed component or a "fail-safe" –designed component? What are the implications for the condition of the timing belt when it is removed?

The timing belt has a scheduled replacement date based on an expected lifetime. If you follow the manufacturer's recommendation, the belt will be removed from service at this mileage level regardless of its condition, so it is possible that it may be on the verge of failure (although this is unlikely because of a safety factor used in the lifetime calculation) or it may still be in excellent shape.

9.10 What is the underlying assumption behind the Palmgren-Minor damage law that may not always be correct?

The Palmgren-Miner cumulative damage law assumes that every stress level contributes to the total lifetime independent of the other stresses experienced by the component. This *assumption makes it possible to add fractional lifetimes in a simple way to predict overall lifetime. However, this assumption of independence may not always be accurate — sometimes the order in which the stresses are applied alters the total lifetime because different stress levels affect the crack nucleation time to different degrees.*

9.11 Given that the 10^6-cycle fatigue limit is often approximately half of the tensile strength of a metal, it would appear to be good design practice to use a material with as high a tensile strength as possible to maximize fatigue resistance. What is the potential shortcoming of this approach?

Fracture toughness decreases and environmental sensitivity increases with increasing tensile strength, so use of a high tensile strength material may introduce problems other than fatigue failure (depending on the application).

9.12 List three general categories of surface treatment that can increase fatigue life, and provide one example of a specific process for each category.

Surface treatments that extend fatigue life include:
Mechanical treatments (shot peening, cold rolling, grinding, and polishing)
Thermal treatments (flame and induction hardening)
Surface coatings (case hardening, nitriding, and plating)

9.13 Polymer fatigue failures may include significant heating. Why is this more of a problem for polymers than for metals?

Polymers often exhibit viscoelastic behavior, so repeated loading and unloading cycles may dissipate significant energy. The amount of anelastic dissipation in metals is usually quite small. Also, polymers typically have lower thermal conductivity than metals, so heat generated in an area with a stress concentration will not be transported away as efficiently for polymers. Finally, such heating can raise the component temperature $>T_g$ wherein component failure will occur by melting.

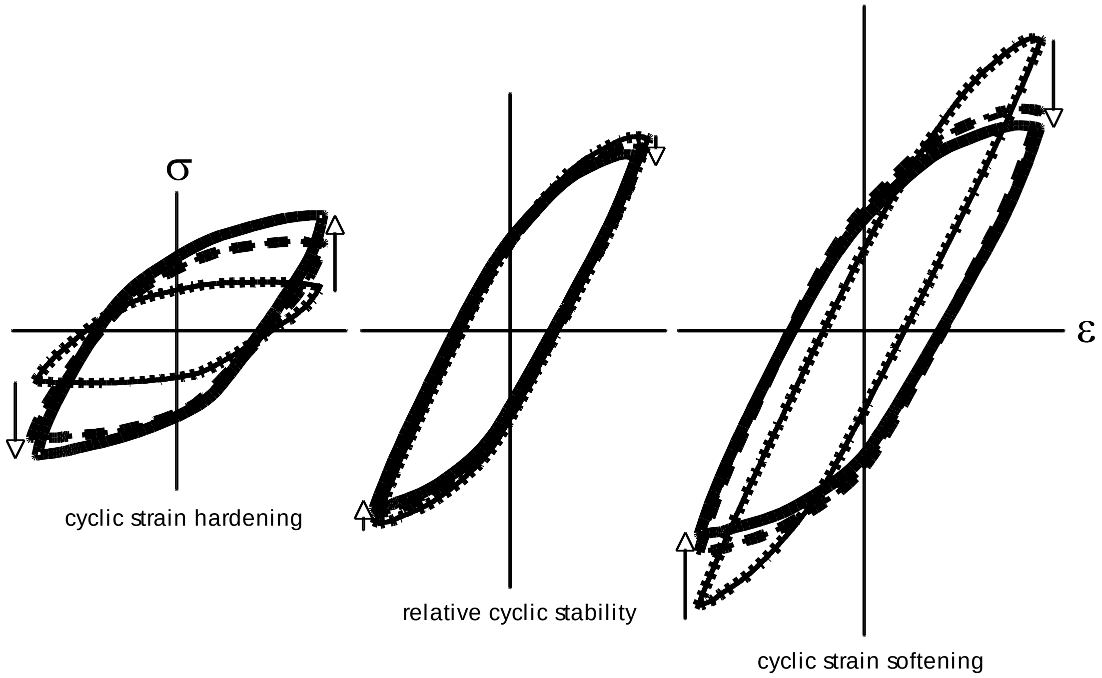
9.14 Explain why the fatigue life (N_f) of a polymer specimen should decrease with increasing frequency.

High frequency means lots of hysteretic dissipation and little thermal transport during a given period of time. This can lead to significant heating and an accompanying decrease in the fatigue lifetime.

9.15 Why is fatigue generally less of a problem with ceramics and glasses than with metals and polymers?

In general, fatigue damage will occur only when cyclic plastic strains are generated. Since ceramics and glasses typically undergo very little plastic deformation, less fatigue damage is expected to occur.

9.16 Sketch a simplified version of Fig. 9.34 that shows only the first, last, and next to last numbered stress-strain loop for each case. Indicate the change in stress from one loop to the next so it is clear what the trend is with regard to cycle-dependent stress in each case.



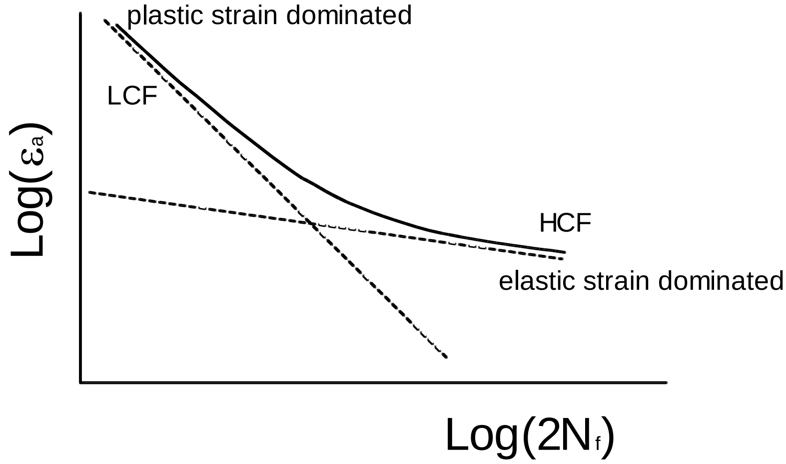
9.17 If you have a material that is *initially hard and strong*, would you expect it to cyclically harden or soften? What would be a way of characterizing *how* strong it must be initially to make your answer a bit more quantitative?

Initially hard and strong materials will generally cyclically strain soften (while initially soft materials will harden). A way of quantifying this is to look at the ratio of monotonic ultimate strength (σ_{ult}) to 0.2% offset yield strength (σ_{ys}). When $\sigma_{ult}/\sigma_{ys} > 1.4$, the material will harden.

9.18 What trend exists between the stacking fault energy of a metal and the rate at which is will cyclically strain harden or soften? Give an example of a common high SFE metal and a common low SFE metal.

High stacking fault energy metals have highly mobile dislocations that can rearrange quickly to rapidly soften or harden the material during cyclic loading. Low stacking fault energy metals have many dislocations that cannot easily cross-slip, and so cannot easily rearrange into a harder or soften configuration. The rate of change is therefore slower for low stacking fault energy metals. Examples of a high SFE material are Al and possibly Cu; examples of a low SFE material are brass and stainless steel.

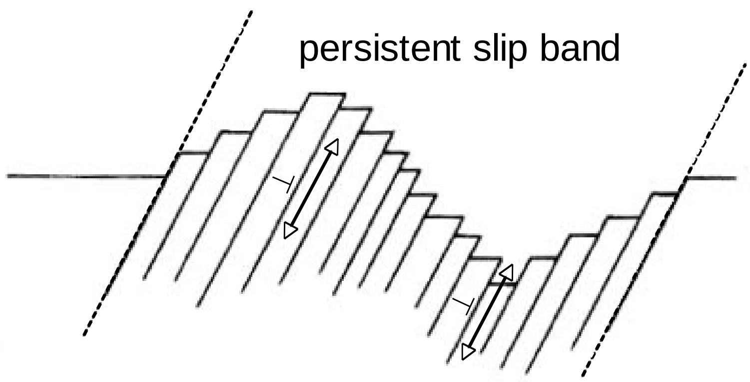
9.19 Sketch a representative ε-N diagram, indicating the regions often called the Low Cycle and High Cycle Fatigue regimes.



9.20 What is the purpose of "rainflow counting"?

The rainflow method is a way to simplify random loading cycles of varying amplitude so that they can be modeled using constant amplitude loading events that are better suited to laboratory and computational study.

9.21 Sketch a persistent slip band, define what it is, and explain its role in fatigue.



Practice

9.22 Two investigators independently reported fatigue test results for Zeusalloy 300. Both reported their data in the form of σ – N curves for notched bars. The basic difference between the two results was that Investigator I reported inferior behavior of the material compared with the results of Investigator II (i.e., lower strength for a given fatigue life) but encountered much less scatter. Can you offer a possible explanation for this observation? Describe the macroscopic fracture surface appearance for the two sets of test bars.

It would appear that Investigator I used test samples with sharper notches as compared with those used by Investigator II. As such, N_i would be lower along with N_T for the case of Investigator I's data set. At low stress levels, both sets of data would show the formation of a *single crack whereas at higher stress levels, there would be many more ratchet lines on the fracture surfaces of Investigator I as compared with Investigator II.*

9.23 Two different polymeric materials were evaluated to determine their respective fatigue endurance behavior. Both materials were tested separately in laboratory air and in flowing water. (Water was selected as a suitable liquid test environment since neither polymer was adversely affected by its presence.) Polymer A showed similar *S-N* plots in the two environments whereas Polymer B revealed markedly different results. Speculate as to which environment was associated with the superior fatigue response in Polymer B and characterize the structure and mechanical response of Polymers A and B.

We may speculate that Polymer A was of the thermoset type and Polymer B was of a thermoplastic type. Since thermoplastic polymers are subject to hysteretic heating during cyclic loading, it follows that Polymer B samples would heat up, thereby leading to inferior fatigue behavior. That would most likely be the case when samples were tested in air; since a water environment would provide for much better heat conduction than in air, the water tests would result in superior fatigue performance. Assuming that Polymer B were of the thermoset variety, little heating would be expected. As such, changes in the test environment would have minimal effect on the thermoset samples' fatigue response.

9.24 For a steel alloy with a tensile strength of 1000 MPa, estimate the fatigue strength amplitude for this material when the mean stress is 200 MPa. Note that you must make an estimate of the fatigue strength using Fig. 9.15 in order to proceed.

We estimate the fatigue strength of the steel alloy (where $\sigma_m = 0$) to be 500 MPa, corresponding to 50% of the tensile strength. Then, using the Goodman relation,

$$\sigma_a = \sigma_{fat}\left(1 - \frac{\sigma_m}{\sigma_{ts}}\right) = 500\left(1 - \frac{200}{1000}\right) = 400\ MPa$$

9.25 Tensile and fully reversed loading fatigue tests were conducted for a certain steel alloy and revealed the tensile strength and endurance limit to be 1200 and 550 MPa, respectively. If a rod of this material supply were subjected to a static stress of 600 MPa and oscillating stresses whose total range was 700 MPa, would you expect the rod to fail by fatigue processes? Hint: You may want to plot a diagram to aid in presenting your answer.

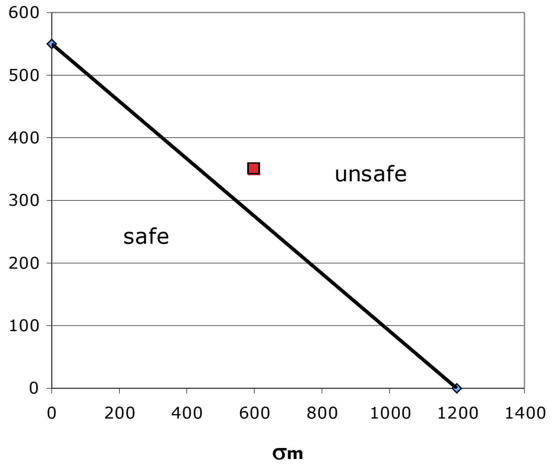
$$\sigma_{fat} = 550\ MPa, \sigma_{TS} = 1200\ MPa, \sigma_m = 600\ MPa, \Delta\sigma = 700\ MPa, and\ \sigma_a = 350\ MPa$$

Using the conservative Goodman equation (Eq. 9-5) we see that for a mean stress of 600 MPa the maximum allowed stress amplitude is

$$\sigma_a = \sigma_{fat}(1 - \sigma_m/\sigma_{ts}) = 550(1 - 600/1200) = 275\ MPa$$

Since the actual alternating stress is 350 MPa, we would expect fatigue failure.

Alternatively, this can be seen graphically by plotting the tensile and endurance limit values in order to construct a failure threshold line, then plotting the actual mean stress and stress amplitude. It can be seen from the diagram that the operating condition falls into the unsafe region.



9.26 The fatigue life for a certain alloy at stress levels of σ_1, σ_2, and σ_3 is 10,000, 50,000, and 500,000 cycles, respectively. If a component of this material is subjected to 2500 cycles of σ_1 and 10,000 cycles of σ_2, estimate the remaining lifetime in association with cyclic stresses at a level of σ_3.

We can estimate the component's fatigue lifetime by using the Miner-Palmgren relation where

$$\sum \frac{n}{N} = 1; therefore, \frac{2500}{10000} + \frac{10000}{50000} + \frac{n}{500000} = 1$$

$$0.25 + 0.20 + \frac{n}{500000} = 1$$

$$n = 275000\ cycles$$

9.27 A cylindrical component will be designed with a circumferential notch to accommodate a small snap ring. If the notch is 1 mm deep and has a radius of 0.5 mm, calculate the theoretical and effective stress concentration factors for this component (i) if it is made of annealed stainless steel alloy 405 with a tensile strength of 415 MPa, or (ii) if it is made with cold rolled stainless steel alloy 17-7PH with a tensile strength of 1380 MPa. What conclusion can you draw about plasticity, stress concentrations, and fatigue life?

Using Fig. 9.16 to determine the notch sensitivity for these two alloys with an identical groove, we find that q_{405}≈0.56 and $q_{17\text{-}7}$≈0.89. The definition of q is

$$q = \frac{1}{1 + p/r} = \frac{k_f - 1}{k_t - 1}\ and\ k_t = 2\sqrt{\frac{a}{\rho}}\ so$$

$$k_t = 2\sqrt{\frac{1.0}{0.5}} = 2.83$$

$$k_f = 1 + (k_t - 1)q = 1 + (2.83 - 1)q$$

$$k_{f,405} = 1 + (1.83)(0.56) = 1.025$$

$$k_{f,17-7} = 1 + (1.83)(0.89) = 2.629$$

When plasticity is limited, as in the case of a high TS metal, the actual stress concentration factor is close to that of an ideal elastic material. However, when significant plastic deformation is possible, as is the case for a low TS metal, the effective stress concentration may be much smaller than the ideal elastic value. When this occurs, the effect of a notch may be much smaller than expected from the ideal elastic stress concentration analysis, and the reduction in fatigue life associated with the presence of a notch may therefore be relatively small as well.

9.28 In an effort to determine a material's resistance to fatigue crack initiation, two studies were undertaken. One investigator used a plate sample with a circular hole 1 cm in diameter, and the other investigator used a similar sample with a 4 cm circular hole. In both cases, the definition for fatigue life initiation was taken to be the number of loading cycles necessary to develop a crack 0.25 cm in length. The cyclic lives determined from these two investigations were not in agreement. Why? Based on the results of Dowling, what change in specimen geometry or initiation criteria would you recommend so that both investigators would report similar initiation lives?

From Dowling's studies, the crack length corresponding to crack initiation should be on the order of $r/e \approx 10$ *with that length corresponding to the transition between short and long crack-controlled behavior. In the two cases described,* $r/e \approx 2$ *and 8, respectively. Therefore,* N_i *would be expected to differ in the two cases. The working definition for crack initiation should be reduced in both instances to approximately 0.05 cm if the same specimen designs are to be used again.*

9.29 The tensile strength of copper alloy C71500-H80 (copper-30%Ni) has been measured at 580 MPa. Estimate upper and lower bounds for the endurance limit at 10^8 cycles for this alloy.

Based on Fig. 9.22, the ratio of fatigue strength to tensile strength of wrought copper alloys falls between 0.35 and 0.5. Thus a reasonable range for the endurance limit at 10^8 *cycles for this alloy is:*

$$0.35\sigma_{TS} \le \sigma_{fat} \le 0.5\sigma_{TS}$$
$$0.35(580\,MPa) \le \sigma_{fat} \le 0.5(580\,MPa)$$
$$203\,MPa \le \sigma_{fat} \le 290\,MPa$$

9.30 Calculate the fatigue life of SAE 1015 and 4340 steel tempered at 425°C when the samples experience total strain ranges of 0.05,0.01, and 0.001. Which alloy is best at each of these applied strain ranges?

The fatigue life can be computed from Eq. 9-20 where $\Delta\epsilon_T = 0.05, 0.01, and\ 0.001$*. The solution below also includes fatigue lifetimes for* $\Delta\epsilon_T = 0.02$ *which represents a cyclic amplitude of 0.01 or* $\pm 1\%$*. The appropriate values of* σ'_f *,* ε'_f *, b, and c are taken from Table 9.3a and the cyclic life,* $2N_f$*,* *computed. A simple computer program was written to arrive at $2N_f$ values that satisfy Eq. 9-20 for the various strain values and the material constants chosen for each alloy.*

$\Delta\epsilon_T$	*Steel 1015*	*Steel 4340*
0.050	*338*	*217*
0.020	*1671*	*1940*
0.010	*6444*	*24600*
0.001	1.88×10^8	1.50×10^{14} *(essentially infinite life)*

Note that the softer alloy (1015 steel) has better fatigue resistance at large strains whereas the stronger alloy is superior at small cyclic strain levels. At a strain amplitude of about 1% ($\Delta\epsilon_T = 0.02$), the cyclic life times for the two alloys are comparable and in the range of 2000 cycles (recall Fig. 9.42).

Design

9.31 If the test depicted in Fig. 9.11 had only been carried out for 10^6 cycles, what approximate fatigue limit would have been claimed for 2024-T3 Al being tested? What assumption would this have led to with regard to designing for infinite fatigue life for a component made from this material? Is this a good assumption under some, all, or no conditions?

Although it is difficult to choose a specific number because of the large scatter, a value close to 280 MPa would be reasonable. This would have led to the conclusion that limiting the design stress of a component to less than 280 MPa would be very likely to avoid any fatigue failure problems. Even with the data beyond 10^6 cycles, it can be seen that this fatigue limit assumption would be acceptable for a component that is expected to undergo fewer than a few million cycles. For certain components this would be fine. For longer lifetimes on the order of tens or hundreds of millions of cycles, however, the assumption of a fatigue limit would be a poor one.

9.32 Cu-Ni alloy 71500 is specified for a certain seawater pump component because of its excellent corrosion resistance, particularly where chloride stress-corrosion may be a concern for steel alloys. With a safety factor already included, the component must be designed to last for 10^8 cycles. If the tensile strength of hot-rolled Cu-Ni alloy 71500 is measured at 380 MPa, estimate the maximum alternating stress that the component can withstand in order to achieve this lifetime.

Based on Fig. 9.22, the ratio of fatigue strength to tensile strength of wrought copper alloys falls between 0.35 and 0.5. Thus a conservative estimate for the endurance limit at 10^8 cycles for this alloy is:

$$\sigma_{fat} = 0.35\sigma_{TS} = 0.35(380\,MPa) = 133\,MPa$$

Extend

9.33 What is a bone *stress fracture*, and how does it relate to the content of this chapter?

A so-called stress fracture in bone is actually a fatigue fracture from repetitive stress.

9.34 Find a photograph of a fracture surface associated with a fatigue failure; it must show beach marks and/or ratchet lines. If the image is not already labeled, add labels for the fatigue markings that are visible. Briefly summarize the circumstances behind the failure, and what can be learned from the appearance of the fracture surface.

Answers will vary widely.

CHAPTER 10

Review

10.1 Dye penetrant inspection, eddy-current testing, and radiographic testing are examples of NDT techniques. What is NDT?

NDT stands for Non-Destructive Testing, a group of techniques used to inspect and characterize the properties of materials without causing any damage.

10.2 Assuming that a given component received a thorough proof test and was found to be satisfactory, would you still be concerned about failure of this component after it was placed into service?

Yes, I would still be concerned. Although failure did not occur, based on the initial test conditions of applied stress and any existing flaw size, subsequent component use may cause a flaw to grow to a critical size. At that point, failure would occur.

10.3 As crack length gradually increases due to fatigue, what typically happens to the fatigue crack growth rate?

The crack growth rate most often increases with increasing crack length.

10.4 How does fatigue crack growth rate vary with applied stress level?

Fatigue crack propagation rates increase with increasing stress level.

10.5 How is it that different styles of fracture mechanics specimen, like those described in Appendix B, can be used interchangeably when determine the fatigue properties of a material?

Fatigue behavior (i.e., fatigue crack propagation behavior) is controlled by ΔK, the stress intensity factor range. Since the Y factor in the stress intensity factor equation can account for the influence of test specimen shape, specimens with different shapes can be used to come to identical conclusions about fatigue crack propagation rates for a given material.

10.6 As a fatigue crack grows from its initial length to its final length, does it spend most of its life in the shorter end or the longer end of the crack size range?

Because fatigue crack propagation rate increases with crack length, the majority of the cycles to failure occur while the crack is small. A change in initial crack length can therefore have a large influence on the total fatigue life.

10.7 For the case of a center notched panel that is loaded by concentrated forces acting at the crack surfaces, what would happen to the crack growth rate as the crack size increased?

For this component-load situation, the stress intensity factor at the advancing crack front would decrease with increasing crack length. That is, $K = P/\sqrt{a}$. *Accordingly, the crack growth rate would* <u>*decrease*</u> *with increasing crack length.*

10.8 The elliptical cracks depicted in Fig. 10.5 change aspect ratio as they grow. Why do they become deeper at a greater rate than they become wider?

In Chapter 6 it was explained that the K factor changes with location along an elliptical crack front. Since the fatigue crack growth rate depends on ΔK*, the rate will be greatest where K is the greatest — typically at the deepest point of an elliptical crack front. This causes greater depth change than width change.*

10.9 What is the main difference between the safe-life and fail-safe design philosophies?

Safe-life design requires replacement of parts when a certain fraction of their anticipated lifetime is reached, regardless of the actual condition of the parts. No inspection is required. Fail-safe design is based on inspection to gather information from which the expected lifetime of a component can be reevaluated, and also redundant structural elements that provide protection against catastrophic structural failure should one load-bearing element fail in service.

10.10 What is the macroscopic shape, orientation, and direction of propagation of the fracture surface of a typical fatigue crack growing in a metal component? What can be learned from the shape? If the shape depends on certain factors, be clear about what these are.

When the stress intensity factor range is sufficiently low that plane-strain conditions prevail at the crack tip (i.e., limited plasticity), the surface is fairly flat and perpendicular to the loading direction. The direction of propagation will also be perpendicular to the loading direction. When the stress intensity factor is much larger, plane-stress conditions pertain and the plane of the fracture surface will shift to +/- 45° to the loading axis. The direction of propagation will still be perpendicular to the loading direction. Thus just as in unstable crack growth, the fraction of flat or slant fracture gives information about the amount of crack tip plasticity occurring during fatigue crack propagation relative to the test panel thickness.

10.11 What are two differences between *clamshell/beach markings* and *fatigue striations*?

One key difference is the source (and meaning) of the marks. Beach marks indicate a transition from one period of fatigue crack growth to another, so a part that experienced thousands of cycles may exhibit only a few beach marks, or none at all. Fatigue striations indicate individual load excursions (one per cycle). There can be thousands or even tens of thousands of striations within individual clamshell markings. A second key difference is the scale of the marks. Beach markings are visible to the naked eye, whereas striations require an electron microscope.

10.12 Are *fatigue striations* always visible on a fatigue fracture surface? If not, under what conditions are they likely to be present and under what conditions are they likely to be absent?

No, they are not always present. Striations are most clearly observed on flat surfaces associated with plane-strain conditions; elongated dimples and evidence of abrasion are the dominant fractographic features of plane-stress slant fracture surfaces. Also, it is much easier to find striations on fatigue surfaces in aluminum alloys than in high-strength steels. Striations are usually associated with intermediate ΔK values. Evidence of microvoid coalescence often occurs at high ΔK levels, and a cleavage-like and/or rough faceted appearance dominates in many materials at very low ΔK levels. Finally, certain environmental or service conditions (such as wear between the mating fracture surfaces) can obliterate striations after they form.

10.13 What important quantity (or quantities) can be determined by measuring striation spacing?

Striation spacing can be used to measure the fatigue crack propagation rate (da/dN) at any given position on the fracture surface. From this, an estimate of the ΔK level at that particular point in the fatigue fracture process can be calculated. If the total crack length at the striation measurement point is also known so that the Y factor can be reliably determined, the stress range can also be calculated.

10.14 Is the Paris Power Law (Eq. 10-3) relevant for all conditions of fatigue crack growth?

No, it is only relevant for intermediate ΔK levels. At very low and very high ΔK levels, the growth rate is more highly dependent on ΔK than predicted from the Paris Power Law.

10.15 When does the mean stress level affect fatigue crack propagation rates in metal alloys?

The answer depends on what portion of the da/dN-ΔK regime one is considering. At high ΔK levels, as K_{mean} increases the growth rates will increase since K_{max} rapidly approaches that of K_c where unstable cracking would occur. At intermediate ΔK levels, there is little influence of K_{mean} on the cracking process. At low ΔK levels where closure levels are high as a result of ASTM recommended test procedures, there will be a large influence of K_{mean} on crack growth behavior since the crack will now be closed for much or all of the loading cycle.

10.16 What does the *crack closure model* say about fatigue damage developed during the compressive portion of an R<0 loading cycle? And, what is ΔK_{eff} in the context of this model?

When the crack is closed due to compressive stress, no fatigue damage occurs. This makes it possible to ignore any compressive segments of a loading cycle when predicting the FCP rate. ΔK_{eff} is that portion of the total ΔK for which the crack is open and damage can occur. Thus $\Delta K_{eff} = K_{max} - K_{op}$ where K_{op} is the K value at which the crack transitions from open to closed.

10.17 What discovery regarding K_{op} motivated the developed of the adjusted compliance ratio (ACR) parameter?

It has been found that there is partial crack opening for K values slightly below K_{op}, *and so the definition of* ΔK_{eff} *as* $\Delta K_{eff} = K_{max} - K_{op}$ *somewhat underestimates the stress intensity range over which damage occurs.*

10.18 What is the meaning of ΔK_{th}, and what implications, if any, does this definition have for design?

Threshold value ΔK_{th} *represents a service operating limit below which fatigue damage is highly unlikely. If it is possible to design a structural element such that* $\Delta K \leq \Delta K_{th}$ *then there is never any cause for concern about fatigue failure. However, this is a very limiting design criterion, and so it is rarely used.*

10.19 Under what circumstance could a fatigue crack become non-propagating, even though the component experiences the same stress cyclic levels?

Ordinarily, the ΔK *level increases as crack length increases for a given stress cycle. However, if some factor other than a change to the stress level causes the* ΔK *level to decrease then crack propagation may slow or stop. For example, when a crack emanates from a notch root, it is initially growing from within an elevated stress zone associated with the stress concentration caused by the notch profile. As the crack lengthens it would be expected that the growth rate would increase as well, were it not for the fact that the crack tip has moved into a region where the local stress level associated with the notch has decreased considerably. Therefore, if the influence of the decreasing local stress level is greater than the influence of the lengthening crack, then the overall* ΔK *level will decline and the associated crack growth rate will diminish and/or arrest.*

10.20 If a constant-stress load cycle is periodically interrupted by an anomalously large stress cycle, does the overall fatigue lifetime of the component remain the same, diminish, or increase?

Surprisingly, the occasional large overlap cycle can actually increase overall fatigue life. This occurs when a large plastic zone is created through which the crack must grow before it can resume its normal growth rate. As it passes through the plastic zone, a higher than normal amount of crack closure reduces ΔK_{eff}, *thereby reducing the fatigue crack growth rate.*

10.21 What is the difference between EAC and corrosion fatigue? What role does K_{IEAC} play in both processes?

EAC is Environmentally Assisted Cracking. It involves subcritical crack growth under static load conditions when an aggressive environment is present. When K is above the K_{IEAC} *level for the material there is time-dependent crack growth without any need for cyclic loading. Corrosion fatigue also include an aspect of environmental attack, but the environment influences the fatigue crack growth rate under cyclic loading conditions. An important aspect of this is that most alloys display a significant corrosion fatigue environmental sensitivity even though the tests are conducted with* K_{max} *maintained below the* K_{IEAC} *level for the material.*

10.22 Does metal corrosion fatigue FCP rate change with increasing temperature? If so, does it typically increase or decrease?

Many investigators have found FCP rates to increase with increasing temperature, which would seem logical given this effect of temperature on both anodic and cathodic reaction kinetics. (The latter, however, is not true for the case of H_2 *gas environmental fatigue since hydrogen uptake and trapping fall with increasing temperature.)*

10.23 Is the nominal fatigue crack propagation process similar for all types of metals and engineering plastics? If so, what is the relationship that best describes the process?

Yes, the process is nominally the same and relates the fatigue crack growth rate with the prevailing stress intensity factor range. That relation is of the form
$da/dN = A\Delta K^m$.

10.24 When choosing a polymer for fatigue crack propagation resistance, would you typically select an amorphous thermoplastic, a semicrystalline polymer, or a heavily cross-linked material? Other than crystallinity and cross-linking, what characteristic of a polymer can have a profound effect on FCP rate?

In general, semicrystalline polymers show greater fatigue crack propagation resistance than their amorphous counterparts. Fatigue crack propagation resistance of engineering plastics increases directly with the material's fracture toughness, so strong heavily cross-linked materials tend not to do as well under FCP conditions. However, rubber-toughening can improve FCP resistance, just as it improves toughness. For all three polymer categories, significant improvement in FCP resistance occurs when the molecular weight is increased.

10.25 Do fatigue striations have the same interpretation in polymers as in metals?

Yes, as for metals, polymer fatigue striations are found to correspond to the incremental advance of the crack as a result of one load cycle.

10.26 Which fatigue marker bands are larger in polymeric solids, *fatigue striations* or *discontinuous growth bands* (DGB)? If one were to examine discontinuous growth band markings at a high magnification, what would you expect to see?

DGB bands would be larger than individual striation markings. The size of a DGB is equal to the size of the Dugdale plastic zone dimension, which in turn is related to the magnitude of the stress intensity factor. A close examination of a DGB would reveal a series of microvoids that decrease in size from the beginning to the end of the band.

10.27 How does the crack growth rate-ΔK dependence of ceramic material compare with that of metals? What effect does this have on our ability to predict fatigue life of ceramic components?

A growing literature in ceramic and ceramic composite materials reveals a Paris-type relation between crack growth rate and ΔK*. However, the exponent for ceramics is between 15 and 42 as compared with a range of 2 to 4 for the case of monolithic metal alloys. This means that ceramic materials are much more sensitive to the* ΔK *level than are metals, and it is much more difficult to reliably predict the fatigue life of ceramic components.*

10.28 If crack-tip plasticity is not to blame for ceramic fatigue, what mechanism is responsible?

Some microplasticity and/or microcracking may occur in ceramics and may contribute to fatigue crack growth. However, other mechanisms may play more significant roles (see Fig. 10.78 for a summary). For example, the tip of a crack in many ceramic materials is shielded by crack wake phenomena such as bridging. Cyclic-induced frictional wear of bridging zones and crushing of asperities on interlocking interfaces may be responsible for the major portion of the fatigue degradation process. With progressive wear, crack tip shielding is attenuated, and the effective stress intensity factor increases. This enhances the likelihood of crack propagation over time.

10.29 What is the general effect on FCP rate of adding reinforcing fibers and whiskers to form a composite?

As might be expected, the addition of reinforcing fibers and whiskers leads to a reduction in FCP rates for a given composite material. This results from the reduction in cyclic strain within the matrix and the transfer of cyclic loads to the fibers. Furthermore, as cracking proceeds within the matrix, unbroken fibers remain behind the advancing crack front and restrict crack opening. This crack-tip shielding mechanism, involving fiber bridging, leads to vastly reduced FCP rates.

Practice

10.30 A component was manufactured in 1950, according to best design practices associated with cyclic loading conditions. By 1960, that part was removed from service. What was the probable cause for removal of the part from service?

Since the part was design and manufactured over 60 years ago, damage tolerant design theorizes had not been developed at that time. Instead, the part was likely would have been manufactured, based on safe life considerations. Hence, the part was most likely removed from service when it had experienced a certain number of loading cycles.

10.31 Imagine that you have two cracked components that are identical to one another except that Component A has a preexisting crack that is twice that found in Component B. Does that mean that the fatigue lifetime of Component A will be 50% that of Component B?

No. Since the crack growth rate varies with the power of ΔK*, the life expectancy of component A will have been reduced significantly, relative to that of component B.*

10.32 A 10-cm-square, 20-cm-long extruded bar of 7075-T6511 is hollowed out to form a thin-walled cylinder (closed at one end), 20 cm long with an outer diameter of 9 cm. The cylinder is fitted with a 7-cm-diameter piston designed to increase pressure within the cylinder to 55 MPa.

(y) On one occasion, a malfunction in the system caused an unanticipated pressure surge of unknown magnitude, and the cylinder burst. Examination of the fracture surface revealed a metallurgical defect in the form of an elliptical flaw 0.45 cm long at the inner diameter wall and 0.15 cm deep. This flaw was oriented normal to the hoop stress of the cylinder. Compute the magnitude of the pressure surge responsible for failure. (For mechanical property data see Chapter 7.)

(z) Assume that another cylinder had a similarly oriented surface flaw but with a semicircular (a = 0.15 cm). How many pressure cycles could the cylinder withstand before failure? Assume normal operating conditions for this cylinder and that the material obeys a fatigue crack propagation relation

$$\frac{da}{dn} = 5 \times 10^{-39} (\Delta K)^4$$

where *da/dn* and *ΔK* have the units of m/cyc and $\text{Pa}\sqrt{\text{m}}$, respectively.

$D_0 = 9\ cm;\ D_i = 7\ cm; t = 1\ cm; P(normal) = 55\ MPa; 2c = 0.45\ cm; a = 0.15\ cm$

a) $K_{IC} = 20.9\ MPa\sqrt{m}\ \ and\ \ \sigma_{ys} = 480\ MPa$

$a/2c = 0.15/0.45 = 0.33$

The estimate of Q depends on the ratio of $\sigma/\sigma_{ys} \approx 0.6, Q = 1.65$

$$K_{IC} = 1.1\sigma\sqrt{\pi(a/Q)} \times \sqrt{\sec(\pi a/2t)}$$

$$20.9 = 1.1\sigma\sqrt{\frac{\pi(0.15 \times 10^{-2})}{1.65}} \times \sqrt{\sec\frac{\pi(0.15 \times 10^{-2})}{2(1 \times 10^{-2})}}$$

$\sigma = 350\ MPa;\ \sigma/\sigma_{ys} \approx 0.73$

Therefore, the estimate of Q is reasonably close. Since

$$\sigma_{hoop} = \frac{PD}{2t}$$

$$350 = \frac{P(7)}{2(1)}$$

$P \approx 100\ MPa$

b) Assume a = 0.15 cm (semi-circular flaw)

$$\frac{da}{dN} = 5 \times 10^{-39} \Delta K^4 (units\ are \frac{m}{cycle}\ and Pa\sqrt{m}$$

$$\sigma = \frac{PD}{2t} = \frac{55(7)}{2} = 192.5\ MPa$$

$$K = 1.1\sigma\left(\frac{2}{\pi}\right)\sqrt{\pi a}$$

$$20.9 = 1.1(192.5)\left(\frac{2}{\pi}\right)\sqrt{\pi a}; a = 0.76\ cm$$

Note that such a final crack length would require a finite width correction of approximately 1.65. Therefore, the critical crack length would be less than 0.76 cm. Without going through a more involved calculation, we may estimate a~0.5 cm, which leads to a K level very close to K_{IC}. The finite width calculation is 1.19 when a = 0.5 cm. For simplicity assume the finite width correction to be constant from a = 0.15 to 0.5 cm and equal to 1.1

$$K = 1.1(1.1)(192.5)\left(\frac{2}{\pi}\right)\sqrt{\pi a}$$

$$\frac{da}{dN} = 5 \times 10^{-39}\Delta K^4$$

$$N_f = \frac{1}{(1.21)^4\left(192.5\times10^6\right)^4\left(5\times10^{-39}\right)\left(16/\pi^2\right)}\int_{a_i}^{a_f}\frac{da}{a^2}$$

When a_i=0.0015 m and a_f=0.005 m, N_f=19.6x10^6 cycles.

10.33 A large steel plate is used in an engineering structure. A radical metallurgy graduate student intent on destroying this component decides to cut a very sharp notch in the edge of the plate (perpendicular to the applied loading direction). If he walks away from the scene of his dastardly deed at a rate of 5 km/h, how far away will he get by the time his plan succeeds? Here are hallowed hints for the hunter:

- **(a)** The plate is cyclically loaded uniformly from zero to 80 kN at a frequency of 25 Hz.
- **(b)** The steel plate is 20 cm wide and 0.3 cm thick.
- **(c)** The yield strength is 1400 MPa and the plane-strain fracture toughness is $48\,\text{MPa}\sqrt{\text{m}}$.
- **(d)** The misled metallurgist's mutilating mark was measured to be 1 cm long (through thickness).
- **(e)** A janitor noted, in subsequent eyewitness testimony, that the crack was propagating at a velocity proportional to the square of the crack-tip plastic zone size. (The janitor had just completed a correspondence course entitled "Relevant Observations on the Facts of Life" and was alerted to the need for such critical observations.)
- **(f)** Post-failure fractographic examination revealed the presence of fatigue striations 2.5×10^{-4} mm in width where the crack was 2.5 cm long.

From item e.,

$$\frac{da}{dN} \propto r_y^2 \text{ and } r_y \propto K^2; therefore, \frac{da}{dN} \propto \Delta K^4$$

$$\frac{da}{dN} = C\Delta K^4$$

From item f, striation spacing = 2.5 x 10^{-4} mm when a = 2.5 cm

For plate free to bend

$$K = Y\sigma\sqrt{a};\ Y(from\ Fig. 8-7f) \approx 2.2$$

When a = 2.5 x 10^{-2} m

$$\Delta K = \frac{2.2(80000)}{20(0.3)10^{-4}}\sqrt{2.5 \times 10^{-2}} = 46.4\ MPa\sqrt{m}$$

For plate not free to bend

$$K = Y\sigma\sqrt{a};\ Y(from\ Fig. 8.7b) \approx 1.95$$

$$\Delta K = 46.4\left(\frac{1.95}{2.2}\right) = 41.0\ MPa\sqrt{m}$$

Using the first solution and letting striation spacing $\approx da/dN$ *as estimate*

$$2.5 \times 10^{-7} = C(46.4 \times 10^6)^4$$

$$C \approx 5.4 \times 10^{-38}$$

Let Y be reasonable constant and ≈ 2.1

$$\frac{da}{dN} = (5.4 \times 10^{-38})(2.1)^4\left(\frac{80000}{(20)(0.3)(10^{-4})}\right)^4 a^2$$

$$\frac{da}{dN} = 3.319 \times 10^{-4} a^2$$

$$\int_0^{N_f} dN = \frac{1}{3.319 \times 10^{-4}}\int_{a_0}^{a_f} \frac{da}{a^2}$$

a_f is computed from K_{IC} assuming plane strain conditions exist.

$$t \,\&\, a \geq 2.5\left(\frac{48}{1400}\right)^2 \geq 0.294\,cm; \text{ which is OK.}$$

$$K_{IC} = Y\sigma\sqrt{a}$$

$$48 \times 10^6 = 2.2\left(\frac{80000}{20(0.3)10^{-4}}\right)\sqrt{a}$$

$$a = 2.68\ cm$$

$$N_f = (3 \times 10^3)\left(\frac{1}{1 \times 10^{-2}} - \frac{1}{2.68 \times 10^{-2}}\right) = 1.88 \times 10^5 cycles$$

Since panel was loaded at 25 Hz.

$$time = \frac{1.88 \times 10^5}{25} = 7.52 \times 10^3 seconds = 2.089\ hours$$

Walking at the rate of 5 km/hr, the student would get 10.4 km away.

10.34 If the plate in the previous problem had been 0.15 or 0.6 cm thick, respectively, would the villain have been able to get farther away before his plan succeeded? (Assume that the load on the plate was also adjusted so as to maintain a constant stress.)

If the thickness had been 0.15 cm instead of 0.3 cm, plane stress conditions would have existed. Therefore, $K_C > K_{IC}$ and a_{crit} would have been greater and life longer. The student would have gotten further away.

If t = 0.6 cm, it wouldn't have made any difference relative to life with the thickness of 0.3 cm.

10.35 Estimate the stress intensity factor range corresponding to an observed striation spacing of 10^{-4}mm/cyc in the steel alloy shown in Fig. 10.56. Compare the results you would get when ΔK is determined from the striation data and the *macroscopic* data in the same figure. Also, compute ΔK from Eq. 10-9.

From Fig. 10.56, a striation spacing of $1x10^{-4}$ mm/cycle corresponds to ΔK range of 25-30 MPa$\sqrt{m}$.
From macroscopic data, ΔK range is from 20-30 MPa$\sqrt{m}$.
From Eq. 10-9

$$st.sp = 6\left(\frac{\Delta K}{E}\right)^2$$

$$1 \times 10^{-7} = 6\left(\frac{\Delta K}{205 \times 10^9}\right)$$

$$\Delta K = 26.5\ MPa\sqrt{m}$$

Which is in good agreement with above estimates.

10.36 (a) A material with a plane-strain fracture toughness of $K_{IC} = 55\,\text{MPa}\sqrt{\text{m}}$ has a central crack in a very wide panel. If σ_{ys} = 1380 MPa and the design stress is limited to

50% of that value, compute the maximum allowable fatigue flaw size that can grow during cyclic loading. (Assume that plane-strain conditions prevail.)

(b) If the initial crack had a total crack length of 2.5 mm, how many loading cycles (from zero to the design stress) could the panel endure? Assume that fatigue crack growth rates varied with the stress intensity factor range raised to the fourth power. The proportionality constant may be taken to be 1.1×10^{-39}.

$$\sigma_d = \frac{\sigma_{ys}}{2} = \frac{1380}{2} = 690\ MPa; 2a_0 = 2.5\ mm$$

$$\frac{da}{dN} = (1.10 \times 10^{-39})\Delta K^4$$

$$55 = 690\sqrt{\pi a}; a_f = 0.002\ m$$

$$\frac{da}{dN} = (1.10 \times 10^{-39})(\Delta\sigma)^4\pi^2 a^2$$

$$N_f = \frac{1}{(1.10 \times 10^{-39})(690 \times 10^6)^4\pi^2}\left(\frac{1}{0.00125} - \frac{1}{0.002}\right)$$

$$N_f = 1.22 \times 10^5 cycles$$

10.37 A thin-walled cylinder of a high-strength aluminum alloy ($K_{IC} = 24\,\text{MPa}\sqrt{\text{m}}$) has the following dimensions: length = 20 cm; outer diameter = 9 cm; inner diameter = 7 cm. A semicircular crack of depth *a* = 0.25 cm is discovered on the inner diameter and oriented along a line parallel to the cylinder axis. If the cylinder is repeatedly pressurized, how many pressure cycles could the cylinder withstand before failure? The pressure within the cylinder reaches 75 MPa, and the material obeys a fatigue crack propagation relation of the form

$$\frac{da}{dN} = 5 \times 10^{-39}(\Delta K)^4$$

where *da/dN* and ΔK have the units of m/cycle and $\text{Pa}\sqrt{\text{m}}$, respectively.

Let σ = 75 MPa and a = 0.25 cm (semi-circular flaw)

$$\frac{da}{dN} = (5 \times 10^{-39})\Delta K^4$$

$$\sigma = \frac{PD}{2t} = \frac{75(7)}{2} = 262.5\ MPa$$

$$K = 1.1\sigma\left(\frac{2}{\pi}\right)\sqrt{\pi a}$$

$$24 = (1.1)(262.5)\left(\frac{2}{\pi}\right)\sqrt{\pi a}$$

$$a = 0.0054$$

The finite width correction for this crack would be 1.12, but for simplicity, we ignore it here.

$$N_f = \frac{\pi^2}{(1.1)^4 (262.5 \times 10^6)^4 (16)} \int_{0.0025}^{0.0054} \frac{da}{a^2}$$

$$N_f = 3.8 \times 10^6 \; cycles$$

10.38 A 2-cm-long through thickness crack is discovered in a steel plate. If the plate experiences a stress of 50 MPa that is repeated at a frequency of 30 cpm, how long would it take to grow a crack, corresponding to the design limit where $K_{limit} = K_{Ic}/3$. Assume that $K_{Ic} = 90$ MPa$\sqrt{m}$ and the material possesses a growth rate relation where $da/dN = 4 \times 10^{-37} \Delta K^4$, with da/dN and ΔK being given in units of m/cycle and Pa$\sqrt{m}$, respectively.

The limiting K level is $K_{IC}/3$. Therefore,

$$K_{limit} = \frac{90}{3} = 30\ MPa\sqrt{m}$$

The limiting crack size for an applied stress is

$$30 = \sigma\sqrt{\pi a} = 50\sqrt{\pi a}$$

$$a = \left(\frac{30}{50}\right)^2 \left(\frac{1}{\pi}\right) = 0.115$$

$$N_f = \frac{1}{(4 \times 10^{-37})\pi^2 (50 \times 10^6)^4}\left[\frac{1}{0.01} - \frac{1}{0.115}\right] = 3.7 \times 10^6\ cycles$$

At 30 cpm, the remaining service lifetime is

$$\frac{3.7 \times 10^6 cycles}{30cpm(60)(24)} = 85.7\ days$$

10.39 An 8-cm-square bar of steel is found to contain a 1-mm corner crack, oriented perpendicular to the length of the bar. If an axial stress is applied from 0 to 420 MPa at a frequency of once every 10 minutes, how long will it take for the rod to fracture? The properties of the bar are: $K_{IC} = 90$ MPa$\sqrt{m}$, σ_{ys} = 1500 MPa, and the crack growth relation is $da/dN = 2 \times 10^{-37} \Delta K^4$, with da/dN and ΔK being given in units of m/cycle and Pa$\sqrt{m}$, respectively.

From Chapter 6,

$$K \approx (1.1)^2 \left(\frac{2}{\pi}\right) \sigma\sqrt{\pi a}$$

Failure will occur when a critical crack length is developed such that $K = K_{IC}$

$$K_{IC} = (1.1)^2 \left(\frac{2}{\pi}\right) (420)\sqrt{\pi a_f}$$

$$a_f = \left[\frac{70\sqrt{\pi}}{420(2)(1.1)^2}\right]^2 = 0.015\ m$$

From Eq. 10-6,

$$N_f = \frac{1}{(2\times 10^{-37})\left[(1.1)^2\left(\frac{2}{\sqrt{\pi}}\right)(420\times 10^6)\right]}\left[\frac{1}{0.001} - \frac{1}{0.015}\right]$$

$$N_f = 46.2[933.3] = 43155 \text{ cycles}$$

For one cycle/10 minutes,

$$N_f = \frac{43155\text{cycles}}{6\text{cph} \times \frac{24\text{hr}}{\text{day}}} = 300\ days$$

10.40 The presence of striations on the fatigue fracture surface of an aluminum alloy is used to determine the magnitude of an overload cycle. Striations immediately before the overload have a width of 2×10^{-4} mm, corresponding to 50% crack closure loading conditions; the overload cycle produced a striation width of 10^{-3} mm. What was the magnitude of the overload cycle?

From the Bates and Clark relation (Eq. 10-9), the striation spacing is proportional to ΔK^2. Therefore, the ratio of striation spacing is proportional to the ratio of the square of ΔK levels or the two load levels.

$$\left(\frac{\Delta K_{OL}}{\Delta K_{normal}}\right)^2 \propto 5 \text{ or } \Delta K_{OL} = \sqrt{5}\Delta K_{normal}$$

Taking crack closure into consideration, striation spacing width depends on the square of the effective ΔK level. Accordingly,

$$\Delta K_{OL} = \Delta K_{OL} - \frac{K_{max}}{2} = \sqrt{5}\left(K_{max} - \frac{K_{max}}{2}\right)$$

So that

$$K_{OL} = \frac{K_{max}}{2}\left[\sqrt{5} + 1\right]$$

$K_{OL} = 1.62K_{max}$ or 62% overload

Design

10.41 A certain steel alloy has been chosen for use in a fatigue limiting service application. Experimental test results provide the following mechanical properties for this material:

$K_{ic} = 50\,\text{MPa}\sqrt{\text{m}}$
$\Delta K_{th} = 4\,\text{MPa}\sqrt{\text{m}}$
$\sigma_{ts} = 1000\,MPa$
$da/dN = 4 \times 10^{-37}\,\Delta K^4$

A plate of this material 2 m wide is expected to experience a cyclic stress range of 200 MPa (R = 0.1) during component operation. You must develop a set of guidelines for the inspection and analysis of this component. If no defect is discovered as a result of NDT inspection capable of detecting cracks as small as 1 mm:

(a) Is it safe to say that the user need not worry about either sudden or progressive failure of this plate?

(b) If fatigue failure is a possibility, estimate the minimum service lifetime for this plate to determine the next inspection interval.

a) If one assumes that <u>*no*</u> *crack exists, then sudden failure would not occur since the maximum stress, 222 MPa, is considerably less than the tensile strength*

$$\sigma_{max} = \frac{\Delta\sigma}{1-R} = \frac{200}{1-0.1} = 222\ MPa\ < \sigma_{ts}$$

Also, the alternating stress is much lower than that necessary for fatigue. Therefore, fatigue would not be expected.

However, if a defect were present <u>*with a size just below the NDT resolution limit*</u>*, the entire scenario would have to be re-examined.*

Assume a through-thickness flaw where $2a = 1\ mm$*. Since a/W is very small*

$$K = \sigma\sqrt{\pi a} = 222\sqrt{\pi(0.0005)} = 8.8\ MPa\sqrt{m}$$

Since K<<K_{IC}, sudden failure would not occur. Furthermore,

$$\Delta K = 200\sqrt{\pi(0.0005)} = 7.9\ MPa\sqrt{m}$$

Since $\Delta K > \Delta K_{th}$*, fatigue failure is possible. As a result, we can confidently state that sudden failure will not occur but fatigue failure may occur.*

b) The final crack length must be determined to establish the limits of integration

$$K_{IC} = \sigma_{max}\sqrt{\pi a_f};\ a_f = \left(\frac{50^2}{222}\right)\left(\frac{1}{\pi}\right) = 1.6 \times 10^{-2} m$$

From Eq. 10-6,

$$N_f = \frac{1}{(4 \times 10^{-37})(\pi^2)(\Delta\sigma)^4}\left[\frac{1}{a_0} - \frac{1}{a_f}\right] = 158.3\left[\frac{1}{0.0005} - \frac{1}{0.016}\right] = 3.07 \times 10^5\ cycles$$

So the next inspection interval should be scheduled for some fraction of the time expect to reach that number of cycles.

10.42 Your company designed a structure that includes an aluminum plate for which you specified routine inspection after every 50,000 loading cycles. The NDT procedure that is employed by the operator of the structure possesses a resolution limit of 1 mm. Through a mix-up, the inspection team calibrated their instrument to yield a crack resolution limit of 1 cm. No crack was found on this occasion, but unstable fracture took place following 34,945 additional loading cycles in association with the development of an edge crack, oriented normal to the major stress direction. Injuries to clients of the operator resulted from the failure. The operator is being sued, but is trying to pass the blame onto your design team. Are there grounds for a lawsuit against you for improper design and inspection specifications, or against the operator based on improper inspection procedures? The key stress level fluctuates between 50 and 100 MPa. The properties of the alloy are: $K_{IC} = 30\ \text{MPa}\sqrt{\text{m}}$, σ_{ys} = 550 Mpa, E = 70 GPa, and the crack growth rate relation is given by $da/dN = 5 \times 10^{-35}\ \Delta K^4$, with da/dN and ΔK being given in units of m/cycle and Pa√m, respectively.

This analysis requires that you determine the size of the crack at the time of the last inspection period. To begin, the final crack size is given by

$$K_{IC} = 1.1\sigma\sqrt{\pi a}$$

$$30 = 1.1(100)\sqrt{\pi a}$$

$$a = \left[\frac{30}{(1.1)(100)\sqrt{\pi}}\right]^2 = 0.0237\ m = 23.7\ mm$$

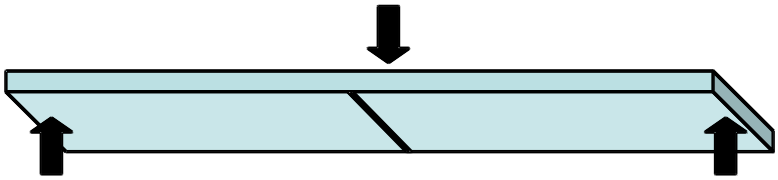
From Eq. 10-6

$$N_f = 34975 = \frac{1}{(5 \times 10^{-35})(1.1)^4\pi^2(50 \times 10^6)^4}\left[\frac{1}{a_0} - \frac{1}{0.0237}\right]$$

$$34975 = 221.5\left[\frac{1}{a_0} - 42.2\right]; a_0 = 0.005\ \text{m}$$

It is seen that the crack size at the time of the last inspection would not have been found, based on the erroneous resolution limit of 0.01 m; however, it would have been discovered and subsequently repaired if the resolution limit was set at the proper level of 0.001 m. Therefore, a lawsuit by the aggrieved party against the operator would be justified, but your design and inspection procedures were adequate.

10.43 A disgruntled former employee has attempted to sabotage a local shipping firm's fleet of trucks. He was caught exiting the company grounds with a hacksaw in hand. He had already cut a long groove across one leaf spring of every truck (as shown below), intending to induce failures once the trucks were out on the road.



A leaf spring is loaded in 3-point bending. The maximum stress experienced when a truck is fully loaded and bouncing along is 250 MPa; under the worst-case scenario, this bouncing occurs at 1 Hz. The material has an elastic modulus of 205 GPa, a yield strength of 593 MPa, and a plane strain fracture toughness value of 54 MPa√m. The deepest cut is 3 mm in a 6 cm thick spring. The support span is 48 cm.

a. The company owner wants to know if it would be safe to drive the trucks temporarily at the maximum load so he can stay in business without risk of immediate failure. Please show clearly if this is the case. A safety factor of 50% is required (i.e., it must survive 1.5x the expected maximum stress).

$$\sigma = 1.5(250\ MPa) = 375\ MPa$$
$$K = Y\sigma\sqrt{\pi a}$$

From Fig. 6.21, and a/W=0.003m/0.060m=0.05, Y is approximately 1.05 without the √π factor, about 1.86 with it included.

$$K = (1.05)(375\ MPa)\sqrt{\pi(0.003m)} = 38\ MPa\sqrt{m}$$

Because $K < K_{IC}$, it is safe to drive the trucks temporarily.

b. Assume that while driving the stress usually alternates between (σ_{MAX} -50 MPa) and σ_{MAX} due to vibration. If this material has fatigue crack propagation parameters of m = 3.0 and A = 1.0 x 10^{-12} (where stress is in MPa and dimensions are in meters), how many load cycles and how many 8-hour days could you expect the leaf spring to survive when the truck is full? (Assume a 5x safety factor here.)

$$\Delta\sigma = 50\ MPa$$

$$N_f = \frac{1}{A\pi^{m/2}\Delta\sigma^m}\int_{a_0}^{a_c}\frac{da}{Y^m a^{m/2}}$$

We need a_c

$$a_c = \left[\frac{K_{IC}}{Y\sigma\sqrt{\pi}}\right]^2 = \left[\frac{54\,MPa\sqrt{m}}{(1.05)(250\,MPa)\sqrt{\pi}}\right]^2 = 0.013m = 13mm$$

Plugging in this value and completing the integration gives $N_f = 23.5 \times 10^6$ cycles. At 1 Hz (1 cycle per second) and driving for 8 hours per day, this is equivalent to 816 days. Even with a generous safety factor of 5x built in, 163 days of safe driving would give plenty of time to arrange for repairs!

Extend

10.44 Find a drawing or a photograph that illustrates the appearance and location of a tear strap used in the fuselage section of an aircraft. Reproduce the image and supply a complete reference for the source.

Answers will vary widely.

10.45 The FAA Federal Airworthiness Requirements (FAR) 25.571 describes the requirements regarding failure resistance of commercial aircraft. In 1964, Amendment 25-0 required that aircraft possess "fail-safe" features. In 1978, Amendment 25-45 added a "damage tolerance" demonstration requirement. What is the difference between the fail-safe design philosophy and the damage tolerance design philosophy?

Fail safe design requires that if any single principal structural element (PSE) of an aircraft should fail, other parts of the structure will have sufficient capacity to keep the aircraft safe until a repair can be made. This is achieved using methods such as the use of multiple load paths (such as "doublers/tear strips") that can support adequate load for safe operation if one path should fail. It also incorporates crack arrest strategies (also accomplished by thickness doublers). An inspection program was expected, but the conditions for establishing the inspection program were based on aircraft service history. Damage tolerance includes these requirements, but also specifically calls for a determination of the most at-risk elements and for a fracture mechanics-based inspection and evaluation plan to ascertain the condition of the at-risk elements after they are put into service. It also allows for the possibility of simultaneous multiple active cracks in principal structural elements.

10.46 Find a recent journal paper that addresses the topic of "ultra high cycle fatigue" and copy its abstract. Also get a copy of the paper so you can extract one figure that has a connection to the contents of this chapter. Write one short paragraph summarizing the point of the figure, making a clear connection to the chapter topics. Give a citation for the paper in proper reference format.

Answers will vary widely.

10.47 Find a recent journal paper that addresses the topic of "fatigue in dental ceramics" or "fatigue in ceramic implants". Write a short review summarizing the point of the paper, making a clear connection to this chapter (and perhaps earlier chapters as well). Provide a copy of the abstract and give a citation for the paper in proper reference format.

Answers will vary widely.

CHAPTER 12

Review

12.1 How recently have the concepts of products liability been introduced into our social awareness?

Recorded history notes the earliest mention of products liability in the Code of Hammurabi that dates back some 3800 years.

12.2 What is the meaning of *caveat emptor*?

The Latin term "caveat emptor" means "buyer beware". That is, a buyer should be careful in their dealings with any vendor, lest they be "taken" by some unscrupulous merchant.

12.3 How is the concept of *stare decisis* applied as it relates to the court's rulings in products liability matters?

A judge may typically rule on a products liability case by conforming to prior decisions rendered in earlier cases. As such, this often leads to predictability in the court's rulings.

12.4 Can a judge render a judgment that is different from earlier rulings of similar cases?

Yes. In such instances, a judge may decide that earlier rulings are in error or do not apply in the present case; also, the judge may determine that a "new" law be established to reflect a new interpretation of the facts. As such, case law is dynamic.

12.5 Do engineers and scientists ever get involved in products liability and product recall matters? If so, in what way?

Absolutely! Engineers and scientists often find themselves providing technical expertise to either plaintiff or defense counsel during the preparation and trial portions of a products liability law suit. Such efforts include examination of all technical factors dealing with the case such as fracture surface details, design parameters, stress and strain levels, and potential environmental factors.

12.6 What is the difference between an express warranty and "puffery" pertaining to a product's characteristics?

An express warranty attests to specific product performance characteristics (e.g., "this auto can accelerate from 0 to 100 kph in 4.5 seconds") that must be achieved. Puffery refers to a dramatic exaggeration of performance specifications—an opinion—as to extraordinary virtues of a product (e.g., "this is the best tasting coffee cake in the world). In this instance, the buyer cannot rely on the validity of such claims.

12.7 In the matter of Gardiner v. Gray, what type of warranty was established?

A lot of fabric, described as being "waste silk", was sold. The buyer subsequently determined that the material he received did not conform to characteristics commonly known in the trade to be that of "waste silk". The buyer sued for damages but the seller argued that there was no "wrong" in that he had not represented the material in the form of an express warranty. Though true, the court found for the plaintiff (the buyer) by arguing that the intent of the transaction was the sale of a material known in the trade to be that of "waste silk". As such, there was an "implied warranty" that the material being transacted was to be "waste silk".

12.8 Assume that you purchased a nationally known lawn mower from your local hardware store. After adding oil and gasoline to the engine and pulling the starter cord, the mower made a loud grinding noise, caught fire, and was then totally destroyed in the flames. In addition, you sustained severe hand burns when attempting to douse the flames. What tort case ruling enables you to sue the manufacturer? In that regard, what concept was struck down when the "law" changed?

McPherson v. Buick Motors Co. The court's ruling struck down the concept of privity, wherein a plaintiff could not sue a manufacturer if the plaintiff had not entered into a formal contract with the company that made the allegedly defective product.

12.9 Define the conditions that reflect negligent behavior.

Negligence is found when: the product contained a defect, reasonable means could have been used to eliminate the defect, the defect caused the injury, and the plaintiff used the product in a reasonable manner.

12.10 Define those conditions associated with a judgment, based on a strict liability interpretation of the law.

- *The product contained an unreasonably dangerous defect.*
- *The defect was under the control of the defendant or that the defect was present when the product left the possession of the defendant.*
- *The plaintiff suffered injuries.*
- *The defect caused the injuries.*

Note that proof of negligence is not required.

12.11 What is the purpose of the Restatement of Torts as they have evolved over the years?

With each different ruling, pertaining to cases involving products liability litigation, the "best" guidelines leading to the "best" opinion becomes increasingly muddled. To bring greater clarity to the law, the American Law Institute (ALI) was incorporated in 1923 for the purpose of "restating" areas of common law, such as that pertaining to products liability.

12.12 List four basic questions of an engineering approach that can help to determine, from a *technical* perspective, when and if to report a "condition" to the CPSC or other similar agency, and whether it is necessary to initiate a product recall.

- *Determination of the Failure Process*
- *Identification of the Affected Product Population*
- *Assessment of Risk Associated with Product Failure*
- *Generation of an Appropriate Corrective Action Plan*

12.13 True or False:

a. All products which present a risk of injury are defective.
b. Hidden hazards and obvious hazards are both hazards, and are, therefore, considered to be equivalent.
c. Root cause and the failure process are not one in the same.

False, False, True.

12.14 List two analytical tools that can assist in evaluating risk associated with a product and explain the differences between these methods.

Failure Modes and Effects Analysis (FMEA) is a qualitative risk assessment framework typically used to prioritize risk before a product is introduced to the marketplace. A traditional FMEA ranks risk in terms of three categories: severity, probability of occurrence, and detection.

A Fault Tree Analysis (FTA) is a quantitative risk assessment methodology whereby the engineer identifies combinations of component or subassembly failures that can result in an incident or 'end point'.

12.15 List proactive considerations that can be incorporated into a product recall prevention and management initiative.

- *Think like a consumer*
- *Test products thoroughly*
- *Ensure adequate traceability*
- *Manage change carefully*

12.16 A returned product that has been involved in a fire has become available for analysis as part of a product recall investigation. Name some of the non-destructive and destructive techniques that could be used in an effort to determine the most likely root cause.

- *X-ray imaging in order to examine the product's interior prior to disassembly.*
- *Examination of visible mechanical damage on the surfaces of the product as well as microscopic examination of any fracture surfaces, if present, to gain an understanding of the loading conditions and the nature of the failure.*
- *Examination of the thermal damage patterns on the product to provide insight into the origin of the heat, smoke, or flame.*

Extend

12.17 Assume that several members of your family were passengers on an ill-fated commercial airliner that crashed upon take-off. Several deaths were reported and many survivors, including your relatives, sustained serious injuries. A subsequent NTSB examination revealed that the airline's maintenance crew had improperly reinstalled engine components that would have enabled the plane to achieve proper lift characteristics so as to obviate the crash. Based solely on the legal opinion set forth by Lord Abinger in the Winterbottom v Wright case, how successful do you believe your relatives and the rest of the passengers would be in receiving compensation for their damages?

Unfortunately, your family would have been unsuccessful in their efforts to be compensated for their injuries and death since there was no privity between your family and neither the plane's manufacturer nor the maintenance crew.

12.18 Today's consumer is provided with information concerning many food products' contents including its caloric content, sugar, sodium, and fat levels. Society has determined that such information is desirable so that the consumer can make a more intelligent decision regarding the health-related appropriateness of such food products. Recalling Eq. 12.1, should non-food product labeling include information pertaining to product failure rates and risk factors to enable the consumer, prior to product purchase, to assess the risk/reward balance for this item?

Conceptually, this idea makes sense were it not for the lack of such information pertaining to these products. In addition, government mandates for the dissemination of such information would be required.

12.19 Using resources provided by the Consumer Product Safety Commission, examine examples of recent product recalls that involve mechanical and/or environmental degradation or damage resulting in product failure. Useful search keywords include *cracking*, *failure*, *fracture*, and *corrosion*. Select one recall example, provide the recall title and a copy of the official hazard description, and briefly explain the cause of the problem in your own words. Finally, propose a solution to the problem that involves a change of materials, manufacturing method, or design.

Responses will vary.

12.20 State your opinion as to the appropriateness of a cost benefit analysis when a case involves personal injury and/or death. What is your reasoning?

Opinions may vary.

12.21 Do you feel that it was appropriate for criminal charges to have been raised in the Pinto crashworthiness case? What is your reasoning?

Opinions may vary.